Mobile Robot Kinematics

Lecture 3

Introduction

* Kinematics studies how mechanical systems behave.
* Kinematics In mobile robotics – understanding of the mechanical behavior of the robot
  * in order to design appropriate mobile robots for tasks
  * In order to control a specific robot

Introduction

- Important kinematic problems for a mobile robot
  - Workspace defines the range of possible poses that the mobile robot can achieve in its environment.
  - Controllability defines possible paths and trajectories in its workspace.
  - Robot dynamics places additional constraints on workspace and trajectory due to mass and force considerations.
    - E.g., a high center of gravity limits the practical turning radius of a fast, car-like robot because of the danger of rolling.
  - A significant challenge: position estimation
    - no direct way to measure a mobile robot's position instantaneously.
    - one must integrate the motion of the robot over time.
    - The inaccuracies of motion estimation due to slippage.
    - Accumulative error

Understanding mobile robot kinematics

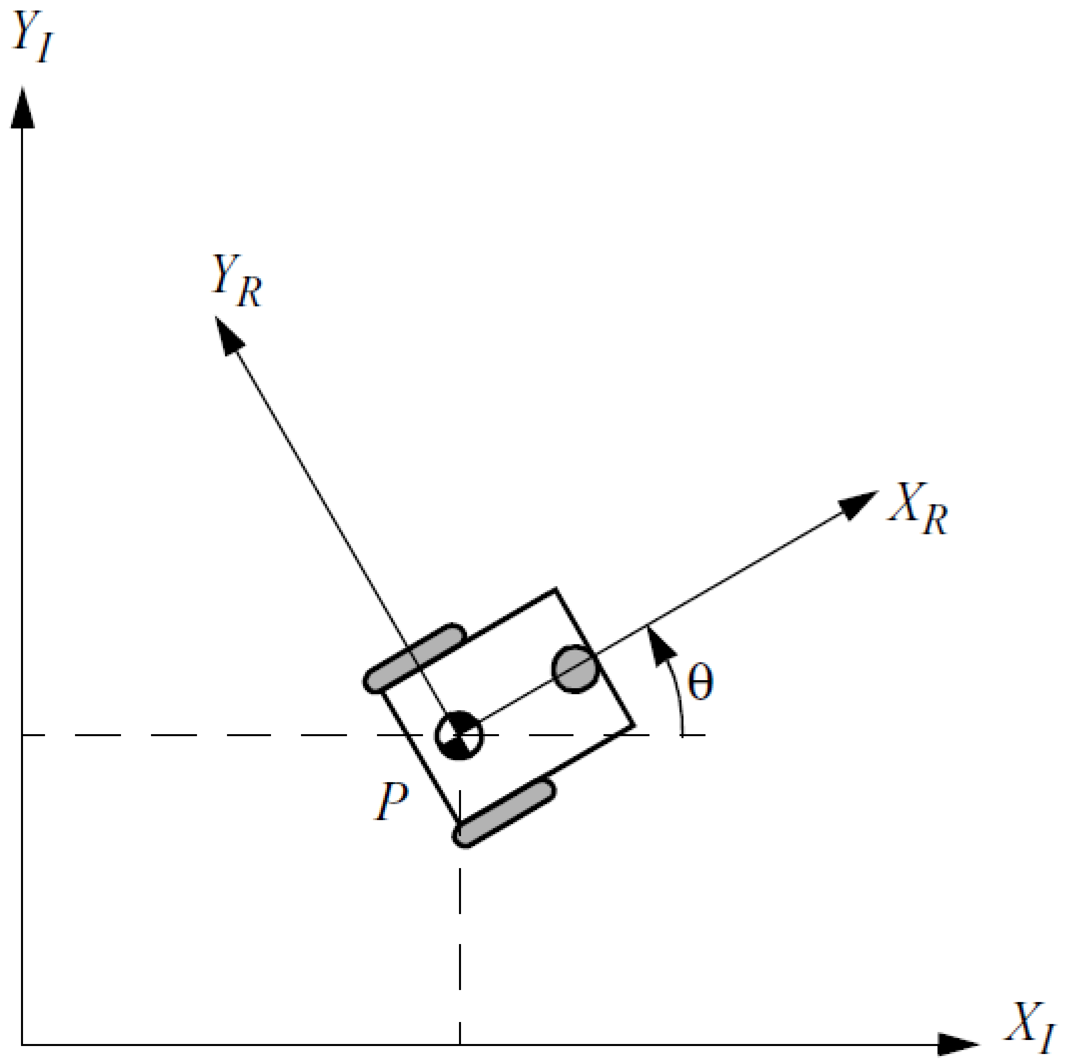
* At the wheel level
  * Understand how each wheel contributes to the robot motion
  * Understand how each wheel imposes constraints on the robot's motion
    * E.g., refusing to skid laterally
* At the robot level
  * Express robot motion in a global reference frame as well as the robot's local reference frame.
  * Construct simple forward kinematic models of motion describing how the robot as a whole moves as a function of its geometry and individual wheel behavior
  * Describe the kinematic constraints of individual wheels, and then combine these kinematic constraints to express the whole robot's kinematic constraints
  * Evaluate the paths and trajectories that define the robot's maneuverability

Kinematic Models and Constraints

- Deriving a model for the whole robot's motion is a bottom-up process.
  - Each individual wheel contributes to the robot's motion and, imposes constraints on robot motion.
  - Wheels are tied together based on robot chassis geometry, and therefore their constraints combine to form constraints on the overall motion of the robot chassis.
  - The forces and constraints of each wheel must be expressed with respect to a clear and consistent reference frame.
  - A clear mapping between global and local frames of reference is required.

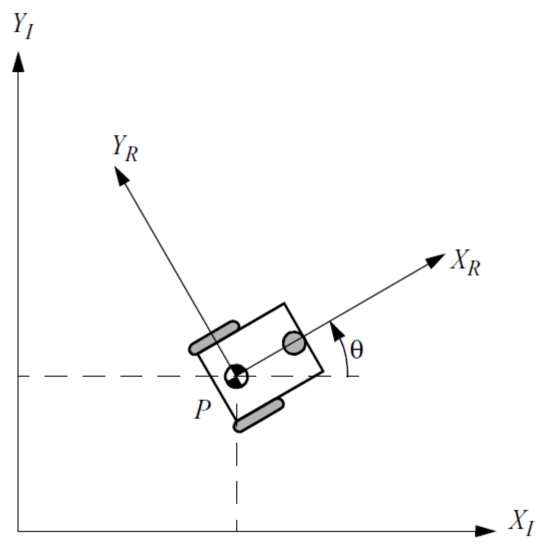
Representing robot position

- We model a mobile robot as a rigid body on wheels, operating on a horizontal plane.
- The total dimensionality of this robot chassis on the plane is three, two for position in the plane and one for orientation along the vertical axis.
  - additional degrees of freedom and flexibility due to the wheel axles, wheel steering joints, wheel castor joints and internal mechanisms are not counted to the robot body's dof.



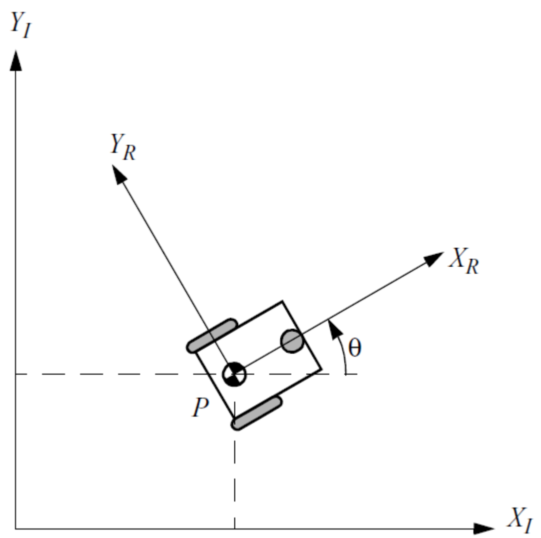
Y_I
Y_R
X_R
θ
P
X_I

* The global reference frame of the plane
  * XI and YI define an arbitrary inertial basis on the plane as the global reference frame from some origin O: {XI,YI}.
  * To specify the position of the robot, choose a point P on the robot chassis as its position reference point.
* The local reference frame of the robot,
  * The basis {XR, YR} defines two axes relative to P on the robot chassis and is thus the robot’s local reference.
  * P is the origin of the local frame.
* The position of P in the global reference frame is specified by coordinates x and y, and the angular difference between the global and local reference frames is given by theta.
* Pose of the robot in the global frame:



$$\xi_I = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

- To describe robot motion in terms of component motions, it will be necessary to map motion along the axes of the global reference frame to motion along the axes of the robot's local reference frame.
  - If the robot moves in its global frame (its own direction) with the velocity, $\dot{x}, \dot{y}, \dot{\theta}$, what is its velocity in the local frame?
  - Why? For control purpose. Control happens in the local frame.
- The mapping is a function of the current pose of the robot. An orthogonal rotation matrix.
- This gives the simplest kinematic model.



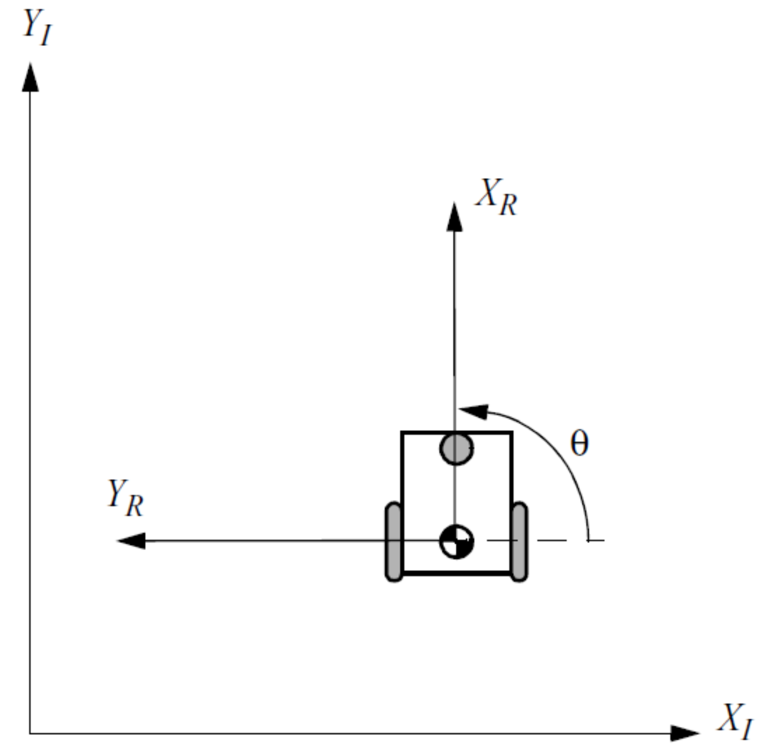
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

* Example:

$$\theta = \frac{\pi}{2}$$

$$R(\frac{\pi}{2}) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



* Velocity in the global frame

$$(\quad \dot{x}, \dot{y}, \dot{\theta} \quad)$$

* Velocity in the local frame

$$\dot{\xi}_R = R(\frac{\pi}{2})\dot{\xi}_I = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\dot{x} \\ \dot{\theta} \end{bmatrix}$$

Homework

* Imagine a point robot follows a circular path. The circle is define as (x-50)^2+(y-50)^2=25^2 in the length unit of m. The robot needs to complete the full circle path in 20 seconds. Program in Matlab to plan the velocity profile for the process, using the point robot kinematic model.