How to Maximize Pectoral Fin Efficiency by Control of Flapping Frequency and Amplitude

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Abstract— **A brute-force testing technique was applied on a robotic controlled-curvature flapping fin propulsor to investigate output thrust and power consumption trends with respect to flapping frequency, amplitude, and gait. It has been discovered that maximum thrust and maximum propulsive efficiency can co-exist, that flapping frequency and amplitude are sometimes interdependent parameters, and that active curvature is more effective than passive curvature. Lastly, the Strouhal number is demonstrated to be ineffective as a tool for describing pectoral fin propulsion.**

Index Terms— pectoral fin, bio-mimetic, efficiency, CFD, kinematics, controlled curvature, Strouhal number, MPF, UUV

I. Introduction

NATURE has always been an inspiration to robotics engineers. Yet not until the last two decades has robotics technology reached a state that it can attempt to accurately imitate the complex behavioral and physiological aspects of living organisms. The study of nature as an inspiration for technological solutions and advancement is called bio-mimetics. Bio-mimetics researchers often argue that natural organisms, through eons of evolution, have fine-tuned their physiology for high energetic efficiency, and that imitating these organisms can grant high efficiencies to bio-mimetic robots.

But overly simplistic bio-mimetic studies can fail to determine what specifically maximizes this biological efficiency. A robot with similar physiology, behaviors, and function to its biological counterpart does not by extension guarantee it will have similar efficiencies. As it is infeasible to imitate a biological organism exactly to the finest detail, a design simplification or deviation can affect efficiency. If an oversimplification is made, all potential efficiency gains from a bio-mimetic design could be lost.

In our previous research [1], a working bio-mimetic pectoral fish fin was built in an attempt to add the agility of fish to unmanned underwater vehicles (UUV). The robotic fin used actively coordinated surface curvature and flapping motions, together called kinematics, to controllably vector average thrust in any generally desired direction [2]. The unknown however was how fin kinematics affected efficiency. What is the effect on propulsive efficiency as flapping frequency and bulk rotation amplitude are varied? Given an infinite number of possible fin kinematics, how can fin flapping frequency and bulk rotation amplitude be actively controlled to both maximize output thrust and minimize power consumption? Can a simple 'rule of thumb' be developed to guide pectoral fin control and design for maximum efficiency?

There is extensive literature offering clues to help define these relationships. When either flapping fin frequency or bulk rotation amplitude is studied independently and increased, the literature shows output thrust increases [1][3]-[8]. However, neither value can increase to infinity, nor are they always independent in terms of output propulsion.

Previous literature approached the pectoral fin flapping efficiency problem in two ways: mathematical analysis, and biological observation – each with limitations. The mathematical analysis method treats the pectoral fin as a rigid plate, using simplified sinusoidal control kinematics, or modeling the fin as an air-foil or propeller; later publications have discounted these interpretations as oversimplified and inaccurate [2][11]-[15]. Other mathematical models of flexible fins have relied on the Strouhal number [14][16][37], a dimensionless value that this paper will demonstrate as ineffective for describing pectoral fin propulsion.

The biological observation method to determine fin efficiency also has several limitations: existing fish species available for study, the number of specimens collected, the variation between specimens collected, the inability to explicitly control biological fin kinematics, and the inability to directly and non-invasively measure both power consumption and output thrust of the biological pectoral fin [17][18][19].

With both computational fluid dynamics (CFD) and a thorough experimental analysis using a robotic fin, this work determines the specific requirements of bio-mimetic pectoral

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fin rotational control that leads to maximum thrust and propulsive efficiency.

This paper is divided into three main sections. First, a background review on our extensive previous work will summarize the robotic fin design, how the fin is controlled, and how thrust and power is experimentally measured. Second, we demonstrate how varying flapping frequency and bulk rotation amplitude affect output thrust, power consumption, and propulsive efficiency. Lastly, mathematical models are built to define and explain energy expenditures, design rules are offered to guide pectoral fin rotation control for maximizing efficiency, and the implications for scaling fin size are discussed.

II. Previous Work And Experimental Setup

The experimental setup has been fully described in previous publications, so only a summary will be presented. See supplied references for in-depth details.

A. Mechanical Design

In previous research a bio-mimetic robotic pectoral fin capable of generating thrust through flapping and active shape deformation was constructed [1]. The design was originally modeled after the well-studied pectoral fin kinematics [7] and structure [20] of the *Gomphosus varius* (bird wrasse), known for relying almost solely on its pectoral fins for both stability and propulsion.

For production of kinematics, the robotic pectoral fin has two important actuator types. As shown inFig. 1 and Fig. 9a, individual micro-servomotors (Futaba S3114, 1.7 kg-cm, 0.09 sec/60°) control fin surface curvature by bending long flexible beams (ribs) embedded within the fin. For rotating the fin about the rotation axis, a single powerful servomotor (Hitec HS-7940 TH coreless digital, 13 kg-cm, 0.07 sec/60°) controls both flapping frequency and bulk rotation amplitude as shown in Fig. 2. Mechanical design was guided by structural optimization [21], computational fluid dynamics simulations [3][22]-[25], and controls simulations based on a pectoral fin propelled UUV [26][27][28]. Further fin design details can be found in [1][2]. This paper focuses solely on improving efficiency through fin bulk rotation control, and not by fin curvature manipulation.

B. Control

Electronic control of the fin and all sensors are coordinated by an Axon ATmega2560 microcontroller running at 16MHz. Individual rib and flapping motions are called kinematics, while a specific set of kinematics designed for a particular task, for example maximum forward thrust, is called a gait. Fig. 1 demonstrates fin shape deformation and bulk rotation over time. Three separate gaits, developed in previous work, were optimized for forward thrust, lift, or reverse thrust generation [2]. These gaits will henceforth be referred to as the forward gait, the lift gait, and the reverse gait. Further fin control details can be found in [1][2][26][27][28].

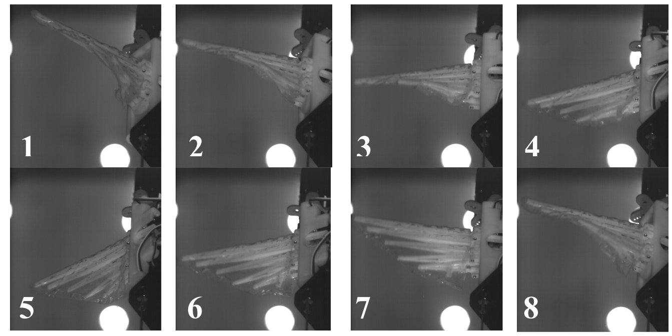

Fig. 1. Example fin surface curvature for an entire flap cycle.

C. Test Setup

As shown in Fig. 2, the fin is mounted on a gantry with two orthogonal torque sensors (model #5350-50, 50 oz-in, by Interface) that measure both lift and thrust produced by the fin. For signal amplification, each sensor uses one LT1102 operational amplifier IC wired as a differential amplifier. Fin forces were determined by measuring torque and dividing by the moment arm length. A waterproof potentiometer (Vishay #P16SNP103MAB15) electrically measured fin angles. A stable power supply set to 5.7V powered the microcontroller and servos, with a separate 10V supply (shifted by +2.5V) powering the torque sensors. Total current was measured by an Allegro 30A ACS715 hall effect-based linear current sensor, while total voltage was measured directly by an ADC on the microcontroller. Current draw for each individual servo was separately measured by a MAX471 current sense amplifier. The water level within the test tank (as described in [2]) was kept ~2.5cm above the fin. Two high-speed cameras mounted at a 90 degree separation allow for 3D measurement and verification of fin kinematics as described in [1][2][30].

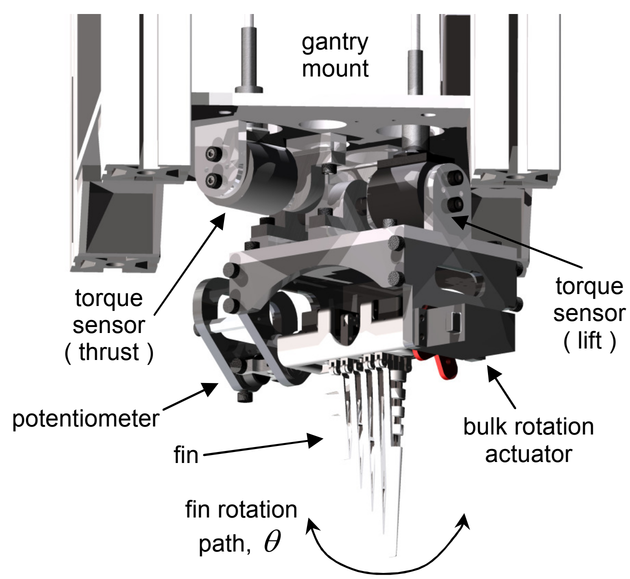

Fig. 2. Experimental force measurement device with fin.

III. Results

Given the difficulty in correctly modeling the mathematically hyper complex physics of the pectoral fin system, we opted for brute-force experimentation to determine how flapping frequency and bulk rotation amplitude affects efficiency. Hundreds of automated experiments were performed varying three parameters: gait, fin flapping frequency, and bulk rotation amplitude. Corresponding thrust, lift, and power consumption were recorded.

Flapping fins inherently have complex three-dimensional velocity fields, complicating the identification of a single velocity value necessary for determining a unit-less efficiency percentage. As such, a fin thrust/power ratio is instead used to compare efficiencies, where a higher ratio signifies a higher efficiency. This efficiency comparison ratio, η_{fin}, is defined as in equation (1), where T_{avg} is the average output thrust and P_{avg} is the average input power in a zero-flow environment.

$$\eta_{fin} \approx \frac{T_{avg}}{P_{avg}} \tag{1}$$

As with previous results in [2], propulsive output in all tested cases reached steady state within a single stroke. As to ensure only steady-state data was considered, measurements were taken over a series of 15 strokes, but only averaged data between the 3rd and 14th stroke were used. Since only averaged data is necessary for a stable pectoral fin propelled UUV [2][26], each experimental run is presented as a single averaged data point. The following plots in sections A-E, consisting of 25 data points per graph, describe lift, thrust, power consumption, and η_{fin} for each gait with respect to fin flapping frequency f and bulk rotation amplitude Θ. Note that section IV will present an in-depth guide to understanding the following results.

A. Forward Gait

Closely matching a sinusoidal motion, the forward gait is designed to produce maximum forward thrust for a UUV. The highest η_{fin} was recorded at the same flapping frequency and amplitude combination as that of maximum thrust. See Fig. 3 for detailed plots.

B. Lift Gait

Closely matching a cupping motion during the downstroke, the lift gait is designed to produce maximum absolute lift (positive or negative) for a UUV with minimal forward thrust. Maximum lift and maximum η_{fin} both occurred at the same flapping frequency and bulk amplitude combination. See Fig. 4 for detailed plots. Results show that lift can be generated without a thrust component.

C. Reverse Gait

Closely matching an inverted Forward gait, the reverse gait is designed to produce maximum negative thrust for a UUV. Maximum negative thrust and maximum η_{fin} both occurred at the same flapping frequency and bulk amplitude combination. See Fig. 5 for detailed plots.

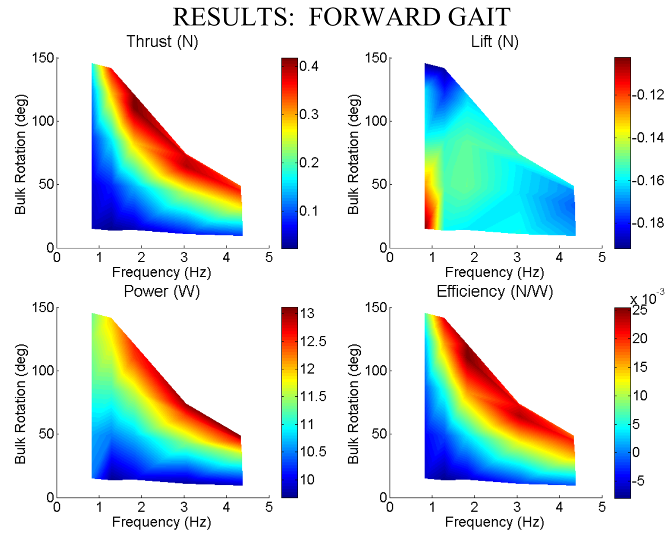

Fig. 3. Forward gait results across all frequency and bulk amplitudes. max thrust is 0.42 N at Θ =111°, f =1.8 Hz, power =12.4 W

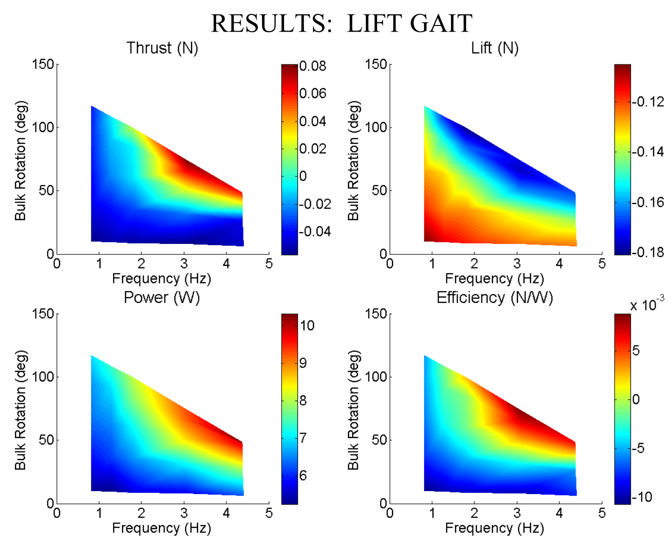

Fig. 4. Lift gait results across all frequency and bulk amplitudes. Max lift is -0.18 N at Θ =100°, f =1.8 Hz, power =8.1 W

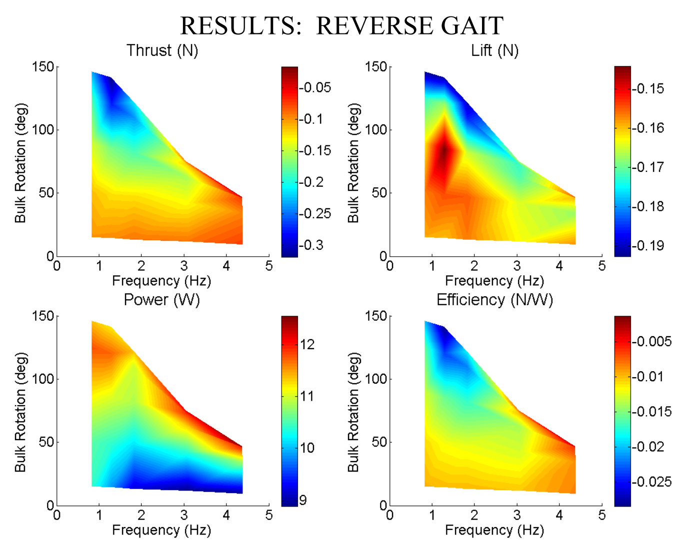

Fig. 5. Reverse gait results across all frequency and bulk amplitudes. max reverse thrust is -0.32 N at Θ =141°, f =1.3 Hz, power =11.2 W

D. *Passive Curvature Gait*

Passive curvature utilizes fewer actuators so therefore has reduced mechanical complexity and power consumption. But it was previously unknown how passive curvature affects propulsive efficiency. As shown in Fig. 6, the fin bulk rotation motor was commanded to flap using forward gait kinematics while the individual rib micro-servo actuators were left unpowered. As with active curvature gaits, both maximum forward thrust and maximum η_{fin} occurred at the same flapping frequency and bulk amplitude combination for the passively flapping fin. Passively deforming thrust was nearly half that of active curvature, yet η_{fin} was almost double. These results experimentally demonstrate that actively-controlled fin curvature produces higher thrust than passive curvature, at the cost of higher power consumption and propulsive efficiency. However, this could be a function of how the specific fin design passively flexes, and possibly not apply to other fins of significant mechanical and kinematic deviation.

E. *Rigid Fin Gait*

Within the literature there is significant research on rigid fin flapping. While a 1:1 comparison cannot be made as our fin does not perform feathering motions, a rigid-fin test can provide clues toward how the geometric shape of a symmetrically flapping fin about a single axis affects thrust.

Our fin was commanded to remain rigid (zero curvature) while the fin bulk rotation motor performed the forward gait. Results are shown in Fig. 7. The rigid fin produced slightly less thrust than the passive fin did, at a lower η_{fin}, with slightly higher power consumption. This power increase was due to the energy the actuators needed to hold the ribs rigid.

IV. Interpretation of Results

The fin test results all carry several important common features. The colored surface plots of each tested gait are bounded by three major regions (see Fig. 8). The left and bottom sides compose the untested region, where early testing [1] determined that very low flapping frequencies and small bulk amplitudes result in ineffective output thrust. As such, no further data in this region was obtained.

The two other regions, the tested and impossible regions as labeled in Fig. 8, are bounded by a curved black line of high significance which defines the limitations of the fin. Given the governing physics of any pectoral fin, it is impossible for fin motions to exceed beyond that curved boundary. This curve is inherent to all flapping actuators, and can be defined as in equation (2) where an increase in fin flapping frequency, f, or bulk rotation amplitude, θ, will result in the decrease of the other.

$$f \propto \frac{1}{\theta_{\max}} \tag{2}$$

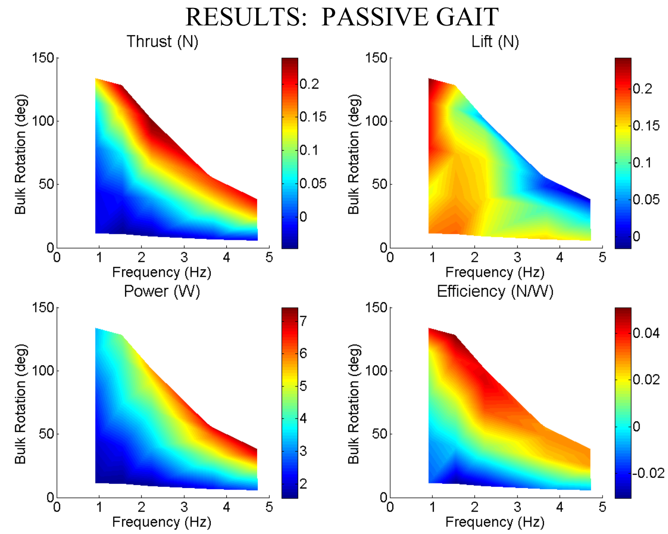

Fig. 6. Passive gait results across all frequency and bulk amplitudes. max thrust is 0.24 N at Θ =102°, f =2.2 Hz, power =5.8 W

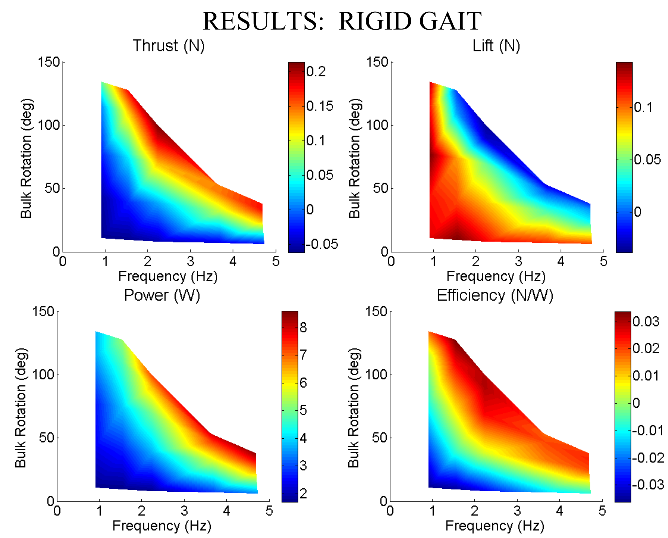

Fig. 7. Rigid gait results across all frequency and bulk amplitudes. max thrust is 0.21 N at Θ =100°, f =2.2 Hz, power =7.2 W

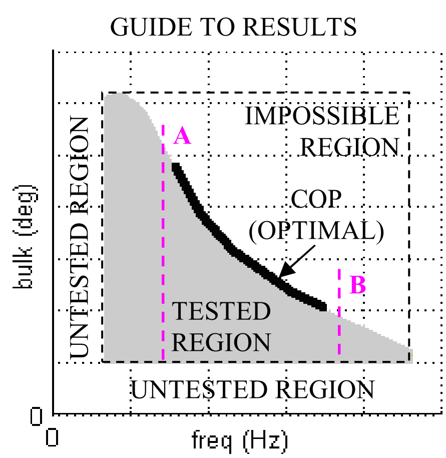

Fig. 8. Guide to Pectoral Fin Brute-Force Testing Results:
Untested Region – poor fin performance
Impossible Region – fin is incapable of this bulk/freq. combo
Tested Region – region where results were obtained
between A and B – desired region for standard fin operation

We define this curved boundary for pectoral fins as the 'curve of optimal performance,' and abbreviate it as COP, as shown in Fig. 8. The COP is bounded by two factors. Any amplitude-frequency combination to the left of dashed line A results in poor output thrust. Any amplitude-frequency combination to the right of dashed line B results in high oscillatory force spikes with the potential to mechanically damage the system.

For all three gaits, including passive curvature and rigid-fin cases, we found the optimal point, O_p, which quantitatively defines the optimal fin flapping frequency and bulk rotation amplitude for both maximum η_{fin} and thrust, to always be located on the COP. This holds great significance in that finding the O_p of any given pectoral fin does not require intense testing. A few short experiments can be performed to locate the COP, and a trend line can be fitted appropriately. By analyzing the η_{fin} trend on that curve, the O_p can be quickly approximated with relatively high accuracy. For all gaits tested, commanding the fin to operate within the Impossible Region will guarantee operation on the COP. While it may be possible to intentionally design a gait where the COP does not exist, there is no evidence to suggest such a gait would be effective.

It should be noted that limited experiments by [29] have come across similar results, where extrapolated data clearly shows the inverse relationship between flapping frequency and bulk rotation amplitude. However, their work did not realize its significance with respect to efficiency.

V. Energy Expenditures

To increase the efficiency of a flapping fin, it must first be understood quantitatively where energy is used in the system. The following sections consider the energy expenditures, offering both experimental data and formulations for the governing physics that define the COP. This analysis will only cover that which defines the bulk rotation, and not the energy spent to create fin surface curvature.

A. Rotational Inertia

Any rotating body with mass has rotational inertia. A flapping actuator, which rotates about its root axis, has rotational inertia partially defining its COP as it must reverse its rotational direction twice during a full fin stroke. Both velocity and energy is lost during each reversal. The flapping fin can be analyzed as a kinetic energy KE problem [31], where energy spent is that which is required to rotate the fin mass about the rotational axis (Fig. 9).

KE is defined as in equation (3), where I is the moment of inertia and ω is rotational velocity.

$$KE = \frac{1}{2} I \cdot \omega^2 \tag{3}$$

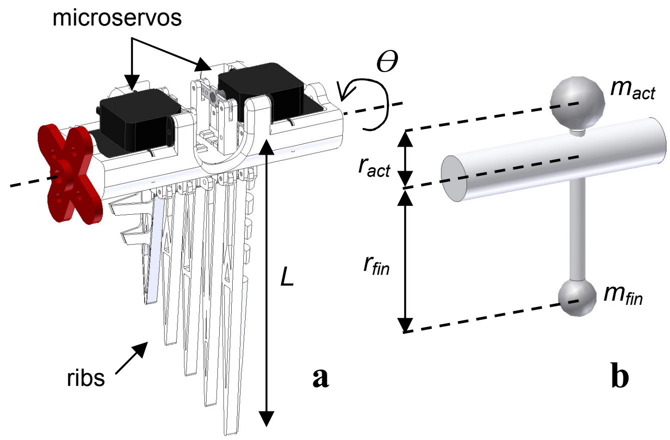

Fig. 9 a) Pectoral fin design, and its b) point mass representation.

To calculate I, the fin is simplified as a point mass m_{fin} at radius r_{fin} from fin axis of rotation as in equation (4) and Fig. 9b. L is fin length, and r_{fin} is approximated as in equation (5).

$$I = m_{fin} r_{fin}^2 \tag{4}$$

$$r_{fin} = \frac{1}{3} L \tag{5}$$

Fin rotational velocity ω can be defined as in equation (6). f is fin flapping frequency, and θ is the maximum bulk rotation amplitude angle in radians.

$$\omega = \frac{d\theta}{dt} = \theta \cdot f \tag{6}$$

Combining equations (3)-(6), we get equation (7) representing the kinetic energy required to flap a pectoral fin. Since the fin must be both accelerated and decelerated during both the up and down strokes, KE_{fin} has been multiplied by 4.

$$KE_{fin} = \frac{2}{9} L^2 \theta^2 f^2 m_{fin} \tag{7}$$

Equation (7) only accounts for the rotational KE of the deformable surface part of the fin, so KE for the rotation of the base and actuators must also be accounted for. Using equations (3), (4), (6), and Fig. 9b, we now get equation (8). r_{act} is defined as the distance from the fin axis of rotation to the actuators center of mass, m_{act}. As with equation (7), KE_{act} has been multiplied by 4.

$$KE_{act} = 2 m_{act} r_{act}^2 \theta^2 f^2 \tag{8}$$

Total fin kinetic energy can be estimated by adding equations (7) and (8). Plugging in typical representative values for masses, lengths, and frequency, the pectoral fin system requires a very insignificant total KE of only ~0.00085W.

B. *Fluidic Energy Losses*

Fluidic loss is the kinetic energy lost to the fluid moving about the pectoral fin. Fluidic losses are very difficult to measure experimentally as all fins have inseparable mechanical losses, too. As such, we took two approaches to determining fluidic loss: computationally and experimentally.

The first approach was done using a 3D unsteady Navier Stokes incompressible computational fluid dynamics solver (CFD) [3][23][32]. The advantage of using CFD over robotic experiments is that it can ignore all but fluidic losses – such as mechanical inefficiencies. Fin kinematics, obtained directly from experiments using our high speed camera system [1][2][30], were modeled within CFD. Each gait was studied at several representative fin flapping frequencies and bulk rotation amplitudes. It was found in all cases, as shown in Table I, that energy lost to the fluid represented less than 2% of total experimentally measured energy consumption.

The second approach was to experimentally change the medium in which the robotic fin flapped. The assumption was that by drastically changing the fluid viscosity, a change in power consumption can be identified representing fluidic losses. The fin performed each gait at several representative fin flapping frequencies and bulk rotation amplitudes in both air and water, and measurements were taken. It was found that not only were the in-air and in-water kinematics nearly identical [1], but no measurable power consumption differences could be identified.

As such, we conclude that fluidic losses are insignificant compared to other energy losses of our pectoral fin system.

TABLE I
EXPERIMENTAL AND CFD COMPUTED POWER CONSUMPTION

Gait	θ	Freq.	Exp.	CFD
Forward	130.3	0.909 Hz	11.5 W	0.1322 W
Forward	64.6	2.208 Hz	14.9 W	0.1159 W
Lift	43.5	3.08 Hz	9.5 W	0.127 W
Reverse	48.3	3.0 Hz	9.5 W	0.168 W
Reverse	125.5	1.0 Hz	11.2 W	0.0987 W

A comparison of mean power consumption computed in CFD and measured experimentally for multiple flapping fin gaits. CFD only measures fluidic energy loses, while experimental measurements account for all losses including inefficiencies. Fluidic energy loss can be considered negligible.

C. *Actuator Loss and Inefficiency*

Because combined kinetic and fluidic energy requirements for fin flapping are an insignificant part of total energy requirements, the only other major energy loss is within the actuators themselves.

To test the energy lost within the actuators, each servo was commanded to perform the required kinematic motions while unconnected to any mechanical systems, i.e. the servos were physically separated from the fin. As no real work was performed, the measured energy drain would approximately represent only the energy wasted within the servo. Much of this energy is lost as heat by the driver circuitry, electro-mechanics of the coils, and gear box efficiency losses. It was experimentally determined that no less than ~89% of all energy was lost in this manner. The remaining spent energy is believed to be used for creating fin curvature, of which is out of scope for this paper.

Therefore, in terms of mechanical design, the single most effective means to improve pectoral fin efficiency is to select actuators with higher efficiencies.

VI. DISCUSSION

The following discussion sections will cover limitations of our experiments, propose implications of our results, discuss further possible research, and declare our current research direction.

A. *On Fluidic Flow*

Our experimental tests were of a fin being actuated without an externally applied flow. CFD was therefore used to confirm that thrust and lift forces remained valid under an external flow scenario [2].

B. *Does Thrust Increase With Frequency and Amplitude?*

Fig. 10 shows frequency compared to output thrust over multiple bulk rotation amplitudes for the forward gait. It shows a general trend that for any given bulk rotation amplitude, as frequency increases, so does output thrust. This is in agreement with previous literature [1][3]-[8].

However, this trend did not hold for the highest flapping frequencies. A similar result occurred for all gaits tested, such as the reverse gait in Fig. 11. Propulsive thrust is not guaranteed to increase with an increase in flapping frequency or bulk rotation amplitude.

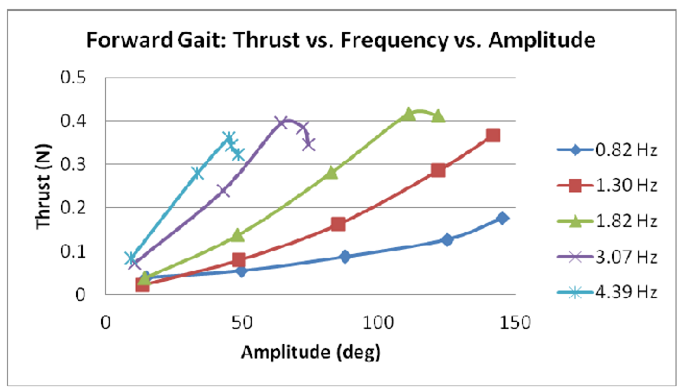

Fig. 10. Thrust output for forward gait

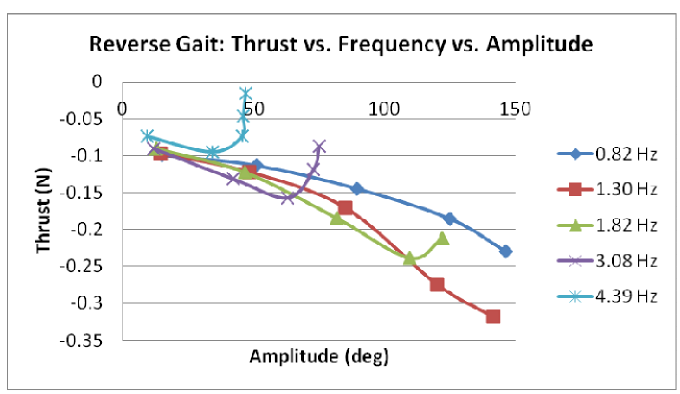

Fig. 11. Thrust output for reverse gait.

C. On Scaling

Although this study did not investigate scaling, the results offer strong clues as to how scaling fin size affects fin efficiency. Given that fluidic and kinematic energy losses were shown to be insignificant, this leaves the scaling of fin efficiency to be solely dominated by the scaling laws of its actuator mechanisms – in this case, DC servos. No literature could be identified which determines power and thrust scaling properties for servomotors.

Nature also offers clues. For biological muscle-based pectoral fins, it has been found that smaller fish have a higher fin flapping frequency [6][33], equivalently smaller fish have a higher 'physiological limit' for flapping frequency [6], and that energy consumption likely scales linearly with biological pectoral fin size [33].

Being a multi-variable problem, deeper research must be done to determine which features dominate efficiency and by how much during fin scaling.

D. The Strouhal Number

The Strouhal number is a dimensionless value often used for describing and predicting the performance of flapping fin designs under varying parameters, with the expectation that difficult to model characteristics would be accounted for. It is defined as in equation (9), where R is the characteristic fin length, f is the fin flapping frequency, and V is defined as the input fluid velocity.

$$St = \frac{R \cdot f}{V} \tag{9}$$

However, the Strouhal number is overly simplified on many accounts. V is poorly defined, as the input fluid velocity across a pectoral fin can be significantly smaller than the output fluid velocity [1], and is neither constant nor evenly distributed across the fin [22]. V is also wrongly assumed to be a single uni-directional vector when in actuality it is a spinning vortex [1][13][14][17][18]. Given these issues, such as for a hovering fin where input flow is minimal and/or unknown, V is often then redefined as the fin tip rotational velocity, V_{tip}.

The length R does not factor in fin shape or surface area, and does not account for fin aspect ratio (AR) – a value well known for influencing fin thrust [1][3][4][12][34]. And though the Strouhal number scales linearly with fin length, there is evidence that suggests fin thrust production scales quadratically with fin length [35].

The fin flapping frequency, f, and the flapping amplitude, θ, do not account for non-symmetric flapping, such as when the upstroke and downstroke have differing completion times. Additionally, the Strouhal number entirely ignores fin surface curvature modulation and its creation of unique kinematics-dependent wake interactions.

Despite a fixed R, f, θ, and V, any thrust vector can be produced by modifying fin surface curvature alone [2]. The Strouhal number also accounts for only positive thrust, yet the lift gait results in Fig. 4 show that just varying θ or f alone can create both positive and negative thrust. This is consistent with preliminary evidence found in [36] and [37], demonstrating that surface curvature modulation alone can have a significant impact on output thrust. Any fin comparisons made with the Strouhal number must be made while holding fin kinematics constant, therefore a separate Strouhal for each fin gait would be required. Comparing similar fins using the Strouhal number cannot be performed using dissimilar kinematics.

As such, the Strouhal number is an ineffective method of predicting pectoral fin performance for all but the most simplified scenarios.

E. On Experimental Automation

Automation of experiments played a key part in this research. The test fin, when including all preliminary experiments, tested 7000+ different gait-frequency-amplitude combinations through 5000+ individual experiments and 90k+ fin flap cycles.

In software, embedded for-loops were used to step through each designed kinematics combination, and then automatically reset all hardware back into starting locations after each experiment. MatLAB and Excel scripts would then process the massive data-dumps into useful summary graphs and tables.

There is one important disadvantage to experimental automation that must be mentioned. Automation is beneficial in that it allows for a greater number of tests to be performed. However, that increase in mechanical cycles resulted in more frequent hardware failures. Materials fatigued and failed, servos designed to work only a limited number of hours burned out, screws slowly came loose, and sensor calibrations shifted. Continual visual inspection of data and hardware was required to identify failures. An automatic failure detection system based on sensor data was not implemented, but is technically feasible given a well understood system.

F. An Explanation for the Biological Refractory Period

In pectoral fin flapping of biological fish, within the flap cycle, exists a yet unexplained 'refractory period' [6][38]-[41]. Between each flap, the pectoral fin pauses for a small period of time. This study offers a possible explanation as to why fish pectoral fins have a refractory period.

As per the results of this study, if a fish desired to travel at maximum speed, it must flap its fins at the maximum physiological rate. This would simultaneously allow for both maximum thrust and maximum propulsive efficiency. But suppose the fish desired to only travel at reduced speed – should it reduce flapping frequency, reduce flapping amplitude, or keep both frequency and amplitude at maximum but add a pause between each flap?

In electro-magnetics, varying the input voltage to a DC motor can vary its rotational speed; however, motors operate at its highest efficiency at only a specific voltage. By controlling the pulse-width of an input square wave at this set voltage, DC motors can then continually operate at maximum efficiency yet still vary speed. This speed control technique is referred to as pulse width modulation, or PWM.

We propose that the refractory period is a biological version of PWM, that it is the means to modulate speed while still retaining maximum pectoral fin efficiency. As large bodies are relatively insensitive to minute oscillating fin forces, a small refractory period would not noticeably oscillate speed or degrade stability [2][26].

As evidenced by the literature, fish velocity has been shown to increase as the refractory period decreases [8]. However, the work by [39] has shown a refractory period only at the highest velocities – evidence that possibly other factors also influence the refractory period. It was suggested by [39] that this difference may contribute to the ability of the surf perch [8] to use pectoral fin locomotion over a larger range of speeds than can the bluegill.

G. Do Fish Pectoral Fins Have a COP?

The literature shows that the cost of transport (COT) of fish pectoral fin propulsion decreases as fish velocity increases [10][42][43], suggesting that pectoral fin propulsive efficiency increases as propulsive thrust increases. This is in agreement with the COP results in this paper.

A COP would also suggest a physical mechanical limitation in pectoral fin propulsion, preventing yet higher thrust. While a well known fact that fish swap from pectoral fin to caudal tail propulsion at higher speeds [6][33][38][39][44][45][46], it was not clear why. Pectoral fin propulsion has a lower cost of transport (COT) and O2 consumption than caudal tail propulsion for many fish species [10][42][44], so this swap is likely not for energetic purposes. Previous literature [9][10] suggested mechanical limitations of pectoral propulsion prevents higher velocities, perhaps due to muscle contraction speed, muscle size, strength of materials, etc. Perhaps only caudal tail propulsion, despite being less efficient, is physiologically capable of operating at these higher velocities.

VII. Conclusion

The effects of fin flapping frequency and bulk rotation amplitude of a pectoral fin were studied both experimentally and computationally. A simple relationship was discovered that maximum thrust and maximum propulsive efficiency can both simultaneously occur given a specific set of fin control parameters. This happens when the actuators reach their physical limitations, where fin flapping frequency and bulk rotation amplitude become inversely related. Fin propulsive efficiency was determined to be dominated significantly by actuator selection. The active curvature fin produced nearly double the thrust of the passive curvature fin, at the cost of higher power consumption and half the propulsive efficiency. The Strouhal number was shown to be ineffective for describing pectoral fin propulsion.

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