A COMPREHENSIVE ALLOMETRIC ANALYSIS OF BIO-MIMETIC MPF-TYPE UUVS

John S. Palmisano, Marius Pruessner, and Jason D. Geder

***Abstract*— An extensive literature review was performed to create a comprehensive allometric analysis of all published bio-mimetic unmanned underwater vehicles (UUV) that employ median and paired fin (MPF; non-caudal, non-body undulating) type propulsion. Informative plots comparing UUV velocity, flapping frequency, mass, length, and fin arrangement are given. Trends offer strong evidence towards generalized scaling laws relating MPF-type UUV velocity, mass, and length.**

Index Terms— pectoral fin, bio-mimetic, allometry, MPF, BCF, UUV, robot

I. INTRODUCTION

FISH propulsion is divided into two categories: MPF and BCF. Median and paired fins (MPF) type propulsion relies on fins located on the sides and/or top of the body, such as pectoral (side) fins. Body and caudal fin (BCF) type propulsion relies on body undulations and/or the caudal (tail) fin [1][2].

While there are advantages to both MPF and BCF type propulsion, this paper will explain why the MPF type has significant advantages over BCF that should be considered when designing UUVs. A plethora of literature in the last ~12 years has been produced on bio-mimetic/inspired MPF-type UUVs – more so than ever before. The goal of those studies was to create a new more effective generation of UUVs. The researchers argued that the living inspirations enjoyed millennia of evolution-driven optimizations to become amazingly effective, and therefore represent a not-yet-achieved gold-standard of robotic capability.

As such, a comparative analysis is now needed to identify not only the more effective designs, but also the specific characteristics which lead to their efficacy. To achieve this goal, we created an extensive database to capture all significant quantitative data on all published designs. By plotting the data and looking for trends, important characteristics necessary for improving future bio-mimetic UUV efficacy can then be extracted.

From a practical standpoint, this analysis will assist engineers in the selection of optimal designs from the literature given specific mission requirements. Additionally, developing generalized scaling laws can assist engineers with the development of scaled test models as well as make predictions towards larger class UUV efficacy.

II. BACKGROUND ON FIN PROPULSION

The characteristics of and differences between MPF and BCF type propulsion are an active area of study for fish biologists. There are tradeoffs between MPF and BCF propulsion, including environmental specialization, speed, efficiency, total power consumption, and maneuverability. Fig. 1 shows examples of both BCF and MPF-type fish.

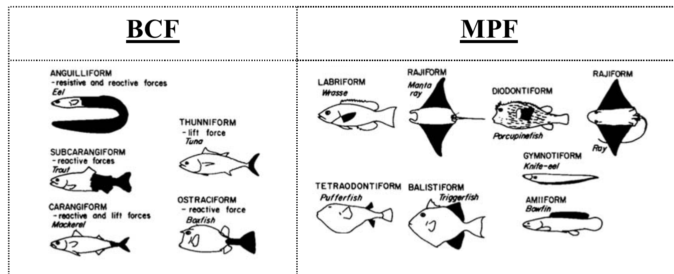

Fig. 1. BCF vs. MPF propulsion comparison (modified reprint from [1]). Fins used for propulsion are colored in black.

Environmental Specialization: MPF is believed to be advantageous for fishes and marine mammals interacting with complex near-shore habitats such as reefs, while BCF swimming is more suited to open-water cruising [2]-[7]. Evidence has also shown that fish populations which employ MPF-based propulsion tend to dominate wave-swept habitats of challenging hydrodynamic conditions [8].

Speed: In nature, fish that rely on the BCF type of propulsion are more effective than the MPF type in terms of maximum forward velocity [4][9][10]. It is important to note that fish prefer to use MPF propulsion at low velocities. But as velocity increases, and the maximum physiological speed limits for MPF propulsion is reached, most fish then transition to BCF to increase velocity yet further. This gait transition from MPF to BCF to increase speed is a well known phenomenon [4][9][11]-[17] and occurs at the highest velocity a fish could likely achieve using MPF-only propulsion [4][9][13][14][15][18].

While for any individual fish the BCF mode will always achieve a higher velocity than the MPF mode, it has been found that some species employing MPF can attain comparable cruising speeds of similarly sized BCF swimmers [7][19]. At low speeds it is believed that MPF propulsion is advantageous over BCF in that MPF swimmers have fewer morphological constraints imposed by streamlining, thereby allowing for a higher degree of variability in form [3][6].

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Beyond the ability to generate higher velocities, BCF has one other advantage: faster braking. BCF has been shown to generate approximately ~3x higher drag than MPF for braking by forming the body into an S-shape [20][21][22], although the actual amount is subject to a scaling effect with respect to body surface area and Reynolds numbers [23].

However, BCF is ineffective at maintaining stability at very low speeds where counteracting external wake disturbances and non-neutral buoyancy are most important. MPF is primarily used for hovering [6][19][24], suggesting MPF is more effective than BCF for low-speed stability.

Efficiency: For fish, the Cost of Transport (CoT) for MPF propulsion decreases with speed [4][9][25], while the CoT for BCF increases with speed [4][9]. CoT of both MPF and BCF is at a minimum during the point of transition between these two modes, suggesting that MPF efficiency is highest when approaching the pectoral fins' maximum morphological constraints. This is in agreement with [26], which shows robotic pectoral fin comparative efficiency (thrust/power ratio) is highest when maximum flapping frequency and amplitude limits are reached. In simplified form, Fig. 2 graphically represents how CoT and efficiency change as MPF transitions to BCF with respect to velocity.

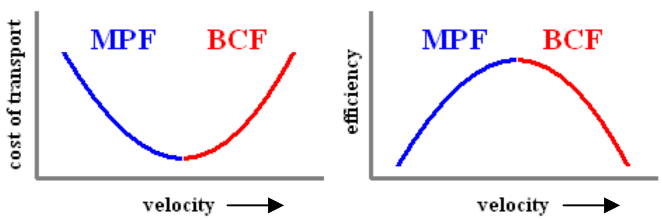

Fig. 2. Fish CoT and efficiency with respect to fish velocity (simplified).

Power Consumption: For fish, as velocity increases, energy consumption also increases [4][9]. This holds true for both MPF and BCF-type propulsion regardless of efficiency. MPF is less power expensive than BCF [16], likely due to the lower fluidic drag associated with lower speeds, the smaller muscle mass required to swim at low speeds [4], reduced drag due to a rigid body [20][27][28], the possibly lower metabolic costs associated with reduced recoil forces in MPF propulsion compared to BCF swimmers [25], and the possibly lower metabolic costs of red (for MPF) muscle over white (for BCF) [4][27][29].

Maneuverability: While BCF-type propulsion has additional degrees of freedom (DoF), it produces only 2 DoF of any real significant thrust. As such, BCF-type propulsion is non-holonomic and therefore has controllability deficiencies. MPF, on the other hand, can vector significant thrust in any direction [30]. Median and paired fins are therefore used as additional control surfaces in tandem with BCF to generate elevating and rolling motions [31].

Another disadvantage of BCF over MPF is when considering reverse thrust. Although some species do use BCF to assist in reverse thrust maneuvers [22], its effectiveness is minimal. To reverse, a BCF fish must first turn its body around using the caudal tail, or scully back using MPF propulsion [6].

Lastly, BCF typically requires some amount of full body undulation – making it difficult to integrate a caudal fin onto a traditional rigid-body UUV.

This paper has focused solely on the MPF type because for the above reasons it is believed to have more utility for use in rigid-body UUVs in complex dynamic environments. Although MPF is inferior to BCF in terms of maximum velocity, MPF propulsion is advantageous in situations requiring low-speed precision manoeuvrability. For example, cluttered near-shore environments, pier systems, hull inspection, mine identification, underwater manipulation of objects, etc., require a stable operating platform.

III. Background on Allometry

Biological systems are extremely complex, spanning from the molecular, cellular, organism, social, and environmental levels, and complicated yet further by the innumerable interdependent components at each level. To broadly draw informative grand-scale generalizations of life, biologists use allometry. Allometry is the well-established method of systematically determining scaling relationships from collected empirical data, such as body size, shape, anatomy, physiology, metabolism, and behaviour.

In allometry, the observed scaling is a simple power law as in equation (1), where X and Y are observables, k is a constant, and b is a scaling exponent.

$$Y = kX^{b} \quad (1)$$

As modern robotic systems become increasingly complex, spanning multiple levels of interdependent components and operating in equally complicated environments, engineers are becoming increasing pressed to similarly develop overall generalizations and scaling laws to inform future system designs. While allometry has been previously used to inform the design of robotic systems [10][32], no study has collected enough empirical data to determine statistically strong scaling relationships across diverse robotic systems. This study determines the allometric power law equations for MPF-type UUV relationships, and then compares them to known allometric relationships of similar aquatic species.

IV. Research Methods

While biologists collect data from living specimens across many species to infer generalizations, the data collected for this study was obtained from an extensive review of all known literature concerning existing UUVs which employ MPF-type bio-mimetic propulsion [33] - [97]. We considered UUV velocity, dimensions (length and width), mass, fin flapping frequency, and fin flapping amplitude. Much of the literature had missing gaps in this data. For example, a publication may report velocity but not vehicle dimensions, or not specify mass, etc. In instances where values were not published or were mathematically contradictory, attempts were made to contact authors to establish and clarify the missing data. When a publication reported multiple velocities, we used the fastest typical value identified. We did not differentiate between dry and submerged weight as most publications did not specify whether a flooded hull design was employed or not. All data were collected into a spreadsheet database using Excel.

The total sample size of this study was 58 bio-mimetic UUVs. Our research did not account for endurance, actuator type, mechanism specifics, turning radius, or cross-sectional shape. Most UUVs were early unrefined laboratory-only prototypes, although several were highly refined and well tested – no distinction was made between either in our research.

For fish, biologists perform a scaling analysis using a single species as the control. Size and shape variation within the species can then be used to make comparisons. But there is rarely more than one of any UUV design. As such, this study was initiated with the hope that overall generalizations towards scaling can be made.

We divided the UUVs into seven MPF-types as listed in Table I. Note that for 2-pect-tail we only considered pectoral fin propulsion – while these systems had caudal tails, pectoral fin propulsive data was collected with the tails deactivated.

TABLE I
DATA REFERENCE CHART

Key	UUV Description
2-pect	two pectoral fins
2-pect-tail	two pectoral fins and a caudal tail
4-fin-turtle	four pectoral fins in turtle-like arrangement
4-fin-other	four pectoral fins in other arrangement
6-pect	six pectoral fins (any arrangement)
manta	has large manta-ray like pectoral fins
oscillating	has two long oscillating surfaces, one on each side

A reference key and short description of each MPF-type UUV analyzed.

V. UUV Allometric Analysis

The following sub-sections comprise of allometric comparisons. Of all data analyzed, only statistically strong relationships are reported in this paper. UUVs which relied on external power sources were not considered for mass comparisons as onboard power comprises a significant percentage of a vehicles' mass. For the below equations, v is for velocity, m is for mass, f is for flapping frequency, and L is for length.

A. Velocity vs. Length

A known velocity-length relationship would allow for the relative prediction of UUV velocity while knowing only its length. The data in Fig. 3 plots velocity vs. length of all MPF-type UUVs analyzed. For manta-type UUVs, a strong linear relationship between the velocity of a UUV and its body length is immediately obvious. This nearly 1:1 relationship is represented in equation (2) and drawn as the diagonal line in Fig. 3.

$$\textit{manta}: \quad L \approx 1.12 * v \quad R^2 = 0.71 \quad (2)$$

The limited data for 4-fin-turtle type UUVs appears to also show similar linearity, and is represented in equation (3).

$$\textit{4-fin-turtle}: \quad L \approx 0.98 * v \quad R^2 = 0.70 \quad (3)$$

Data was too sparse to determine the velocity-length relationship for other UUV types.

For a biological comparison, at maximum fish velocity using only two pectoral fins, an equally linear velocity-length relationship has been identified [14][15][98][99]. There are no published biological studies which compare length and velocity for mantas or turtles.

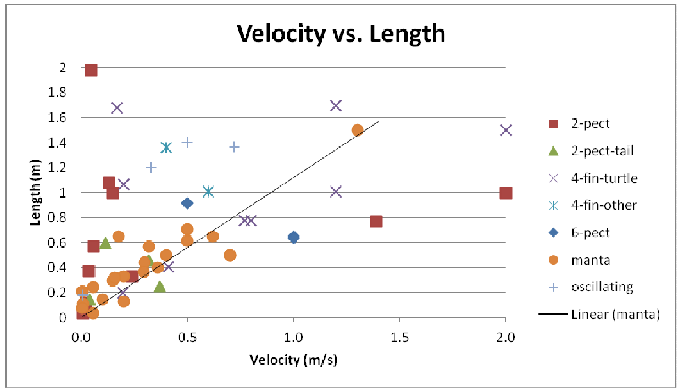

Fig. 3 A comparison of UUV velocity vs. length.

B. Velocity vs. Mass

Mass affects CoT, high-speed maneuverability, and portability of the UUV system. The data in Fig. 4 demonstrates the relationship between the velocity of a MPF-type UUV and its mass. The relationship is exceptionally strong for manta-type UUVs, and is shown as a diagonal line. Equation (4) is the relative linear scaling relationship between UUV mass and velocity when UUV type is not considered. A linear relationship appears exceptionally strong for manta-type UUVs as defined in equation (5), and possibly exists for 4-fin-turtle UUVs as defined in equation (6).

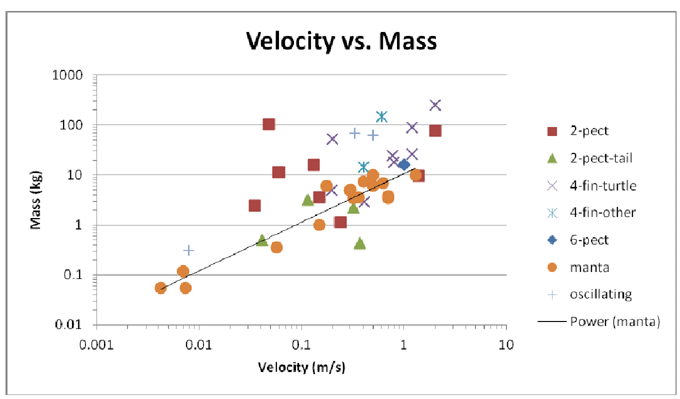

Fig. 4 A comparison of UUV velocity vs. mass.

overall: $m \approx 21.75 * v^{0.94}$ $R^2 = 0.52$ (4)

manta: $m \approx 10.69 * v^{0.97}$ $R^2 = 0.91$ (5)

4-fin-turtle: $m \approx 41.39 * v^{1.06}$ $R^2 = 0.38$ (6)

While there is some velocity-mass relationship data for biological turtles [100], it is insufficient for determining an allometric relationship. There are no published biological studies which compare the velocity and mass of mantas.

For biological surfperch at the critical pectoral-caudal transition velocity, an exponential velocity-mass relationship has been identified [14]. While data was sparse, the relationship is represented by equation (7). Our data for 2-pect and 2-pect-tail designs have a similar relationship; however the low R^2 coefficient of determination is too inconclusive.

surfperch: $v \approx 0.51 * m^{0.17}$ $R^2 = 0.87$ (7)

C. Fin Flapping Frequency

Fin flapping frequency was compared to UUV length, velocity, and mass. However, no identifiable allometric pattern relating to UUV fin flapping frequency was found. This result is inconsistent with the literature in regards to biological allometry, as flapping frequency in nature has allometric relationships.

While the majority of UUVs had a flapping frequency between 0.5Hz and 2Hz, it is unknown whether this range represents an optimal flapping rate for UUVs. The majority of the literature gave no information on how flapping frequency was selected or if it was optimized for any parameter. This range is perhaps a coincidental result of the actuators and mechanisms currently available to engineers at this size and scale, and what gave subjectively reasonable thrust.

1) Flapping Frequency vs. Length

UUV length was compared to fin flapping frequency. As plotted in Fig. 5, no allometric relationship between UUV length and flapping frequency was found. The only trend identified was that most MPF-type UUVs were between 0.25m and 1m in length.

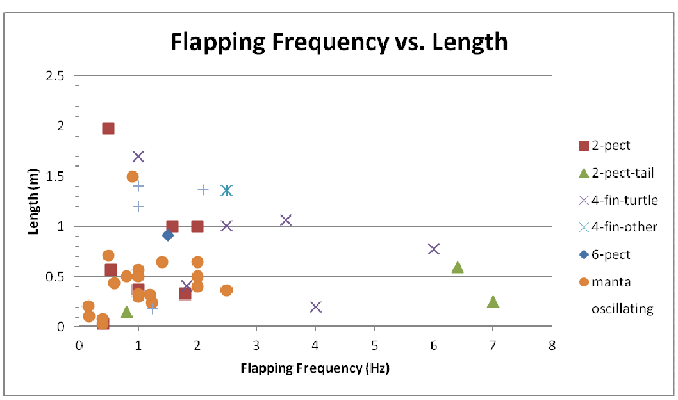

Fig. 5 A comparison of UUV flapping frequency vs. length. No allometric relationship found.

Within the biological literature, [13] showed that longer striped surfperch fish can achieve higher velocities for any equal fin flapping frequency, but the biological allometric relationship is otherwise yet unclear. Both [13] and [15] observed that fish flapping frequency is not related to size at slower speeds, while [15] observed that flapping frequency is significantly dependent on size at higher speeds.

2) Flapping Frequency vs. Velocity

UUV velocity was compared to fin flapping frequency. Similar to Fig. 5, no allometric relationship between UUV velocity and flapping frequency was found. The only identifiable trend is that most MPF-type UUVs have a velocity of less than 0.7 m/s.

Within the biological literature, it was found that the velocity-frequency relationship is linear for parrotfish [4] and the bluegill sunfish [9][101]. Research by [15] and [98], which examined multiple labriform species, showed an approximately linear relationship between velocity and flapping frequency. On the contrary, [13] showed that striped surfperch fish velocity exponentially increases with fin flapping frequency. However, the surfperch velocity-frequency relationship became linear when adjusting the data to remove the fin ‘refractory period’.

3) Flapping Frequency vs. Mass

UUV mass was compared to fin flapping frequency. Similar to Fig. 5, no allometric relationship between UUV mass and flapping frequency was found. The only identifiable trend is that most MPF-type UUVs have a mass between 1 and 11 kg.

Within the biological literature, [102] found a strong relationship between flapping frequency and mass for aquatic species, including birds, fish, and mammals, and is reproduced in equation (8). Although that study did not distinguish between MPF and BCF, it did not seem to affect the results. Regardless, most data points came from MPF-type species. Interestingly, turtles were the only species determined not to follow their determined frequency-mass relationship. Work by [14] also found a frequency-mass relationship for striped surfperch, and is reproduced in equation (9).

$$f \approx 3.56 * m^{-0.29} \quad (8)$$
$$f \approx 3.05 * m^{-0.13} \quad (9)$$

4) Flapping Frequency vs. Flapping Amplitude

As shown in our recent work [26], amplitude and frequency are inversely proportional variables when a flapping fin is producing maximum thrust. As such, it can be predicted that flapping fins of high frequency would have a low flapping amplitude, and fins with high flapping amplitudes would have a low flapping frequency. Fig. 6 plots UUV fin flapping amplitude and fin flapping frequency, confirming what was predicted by [26]. It must be noted that many MPF-type UUVs likely did not experimentally optimize these two parameters to maximize vehicle velocity, and that both frequency and amplitude limits are dependent on actuator and mechanism type. An in-depth analysis on the repercussions of this result can be found in [26].

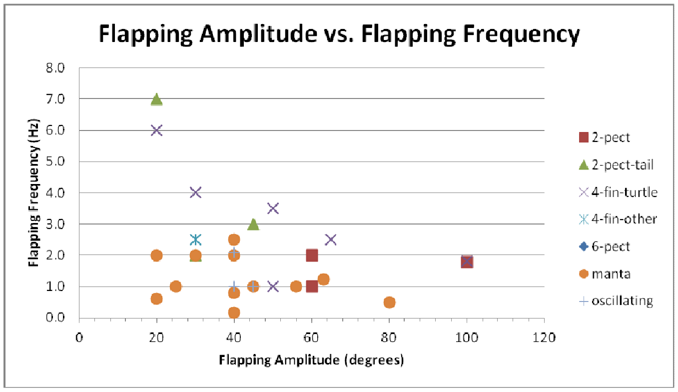

Fig. 6 A comparison of UUV fin flapping amplitude vs. flapping frequency.

D. *Mass vs. Length*

It is a well known fact of biological allometry that mass of an organism scales by the cube of its length, as shown in power law equation (10). To be sure, biological studies prove this mass-length relationship for manta and other BCF/MPF fish types [103]-[106], and also for turtle species [107]-[111]. While the exact constant, k, varies by orders of magnitude between different species and specific fin arrangements, in all of those studies the general trend is that mass scales to the cube of its length.

$$m \approx kL^3 \qquad (10)$$

However a different scaling correlation was found between UUV mass and length, as plotted in Fig. 7.

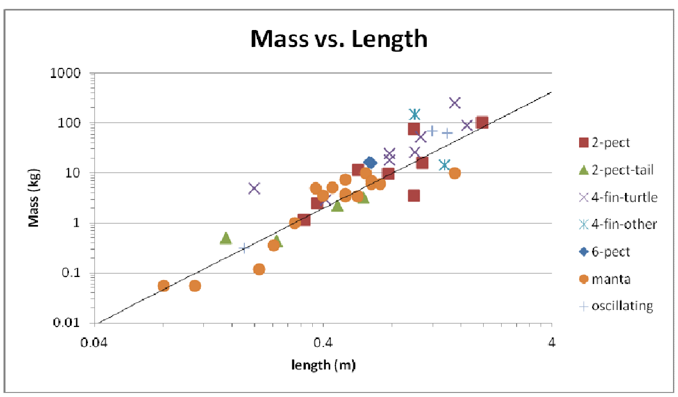

Fig. 7 A comparison of UUV mass vs. length.
Interpolated power curve represents all data points.

While data for manta-type UUVs were sufficient enough to determine the relation as in equation (11), the limited data for other UUV types appear to follow a similar trend. The determined mass-length relationship for the 4-fin-turtle and 2-pect types are defined in equations (12) and (13), respectively.

The overall UUV mass-length relationship, when fin arrangement is not considered, is defined as in equation (14).

manta: $m \approx 16.50 *L^{2.32}$ $R^2 = 0.85$ (11)

4-fin-turtle: $m \approx 40.78 *L^{1.82}$ $R^2 = 0.78$ (12)

2-pect: $m \approx 18.80 *L^{2.19}$ $R^2 = 0.68$ (13)

overall: $m \approx 24.32 *L^{2.43}$ $R^2 = 0.82$ (14)

As demonstrated, the mass of man-made MPF-type UUVs scale closer to the square of its length – contrary to scaling of biological systems. The authors hypothesize several reasons for this result:

1) UUVs do not experience the same scale-independent molecular-chemical and physiological limitations as biological systems do; have different energy storage chemistries and densities; and robotic systems use materials of higher density with different physical properties, such as plastics and metals, as compared to bone and tissues [29].

2) UUVs do not have respiratory, reproductive, or digestive systems. They do not require mechanisms to self-heal, grow, feed, attract mates, defend against predators, hunt prey, etc. Mission operation times for UUVs are typically measured in hours, carrying only mission-specific payloads.

3) UUVs use depth-controlling piston tanks. These tanks are more effective at compressing air than biological swim bladders, thereby affecting volumetric requirements.

4) UUV mass measurements reported in the literature rarely differentiated between dry and wet weight. Some UUVs had flooded hulls, adding additional mass only when underwater.

VI. Reference Data

Fig. 8 through Fig. 13 plot reference data that the reader may use to correlate UUV design to previous figures. The data can assist the reader in selecting an optimal UUV design, given a specific set of UUV mission requirements. UUVs within the charts are grouped together by propulsion type.

A missing bar represents data we were unable to obtain. Mass for UUVs which employed external power sources were intentionally left blank as onboard power typically comprises a significant percentage of a vehicles' mass.

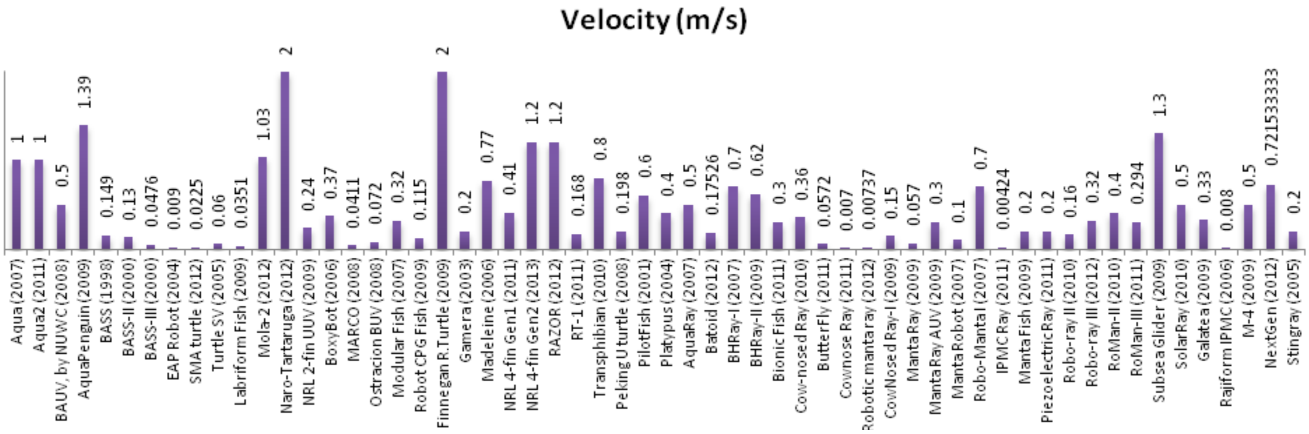

Fig. 8 A reference of UUV velocity data.

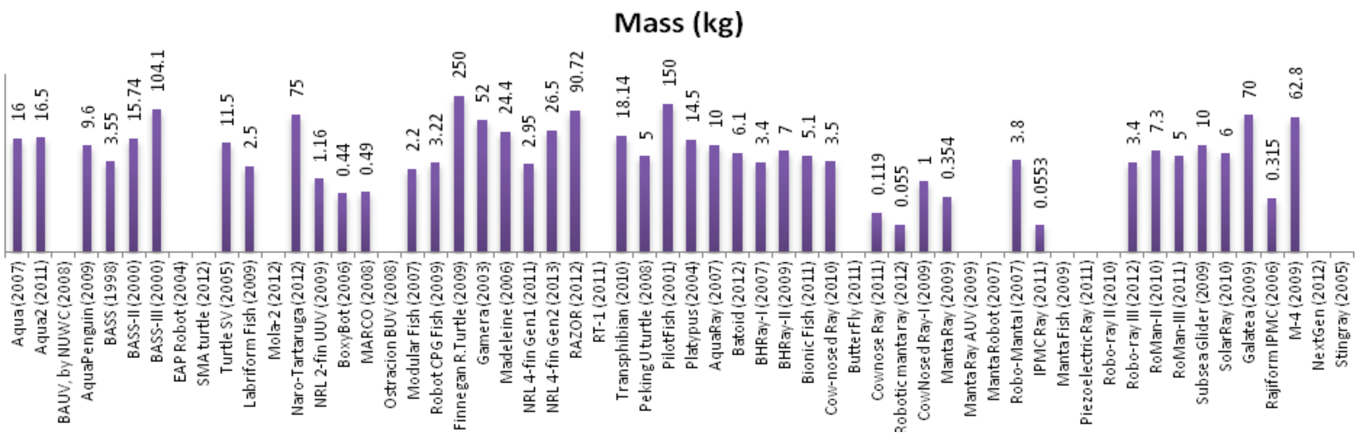

Fig. 9 A reference of UUV mass data, scaled logarithmically. Masses are unlisted for UUVs with external power supplies.

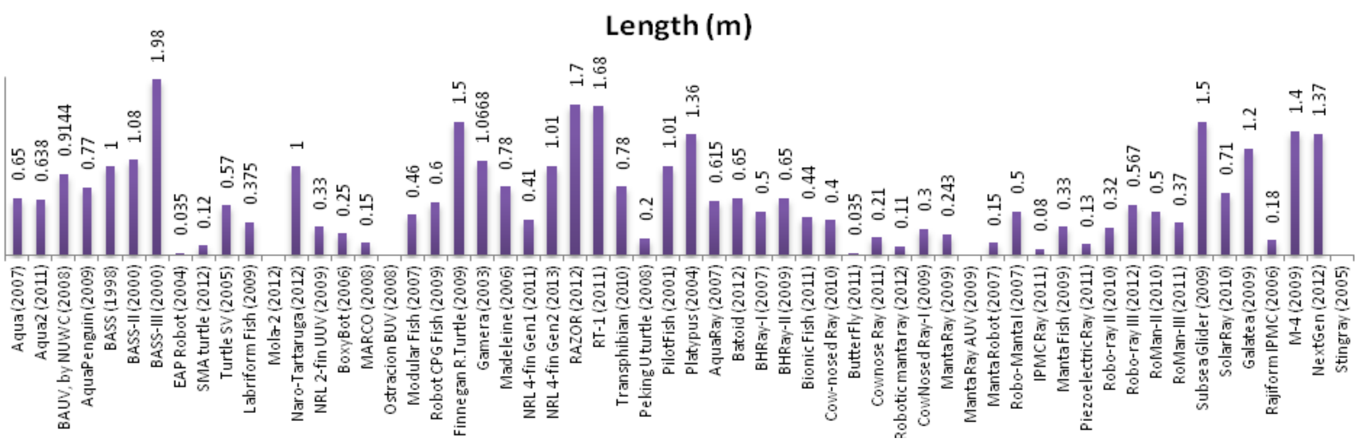

Fig. 10 A reference of UUV length data.

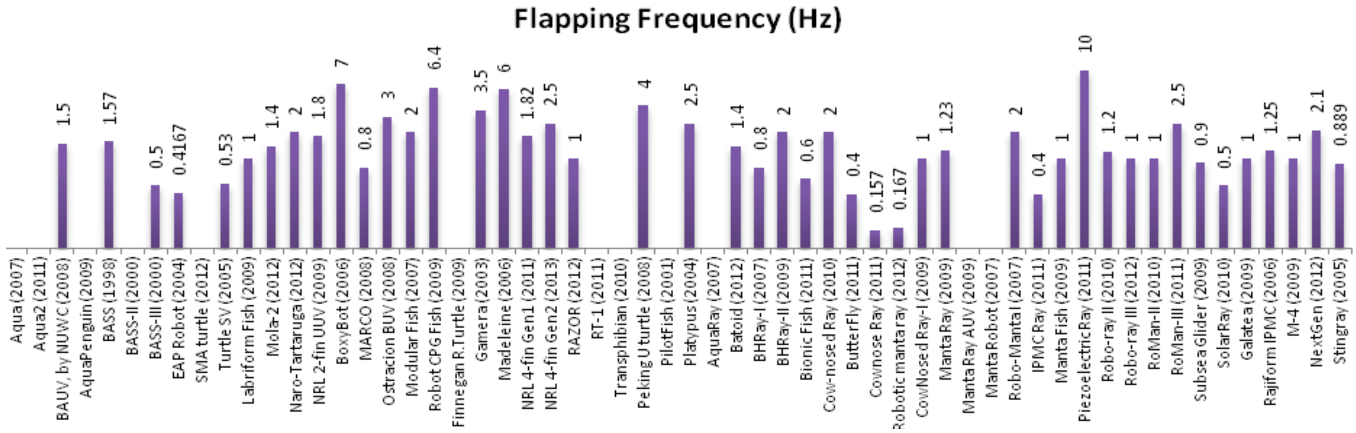

Fig. 11 A reference of UUV fin flapping frequency, scaled logarithmically. Only values determined to give the highest UUV velocity are listed.

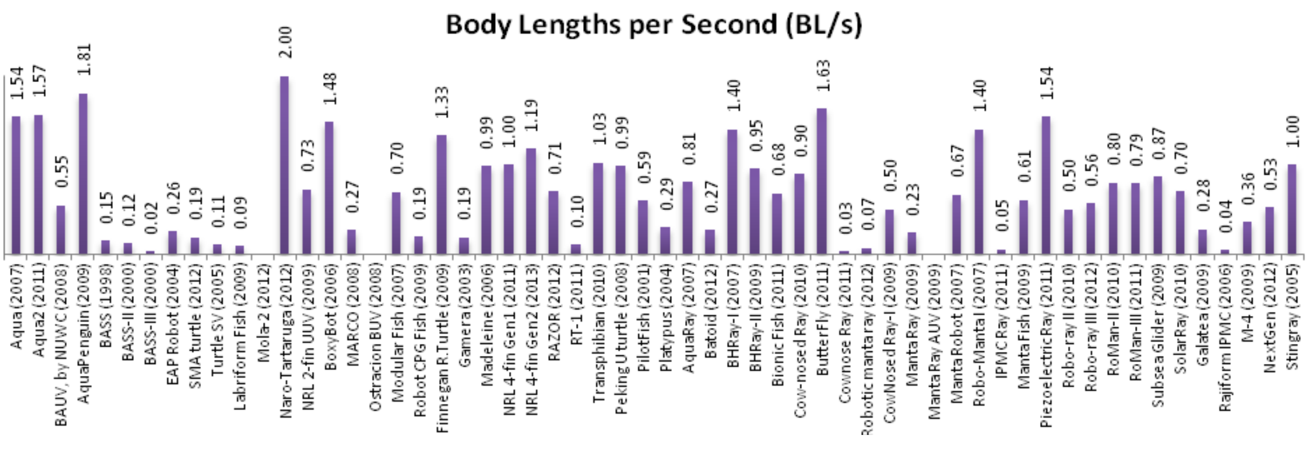

Fig. 12 A reference of UUV body lengths per second.

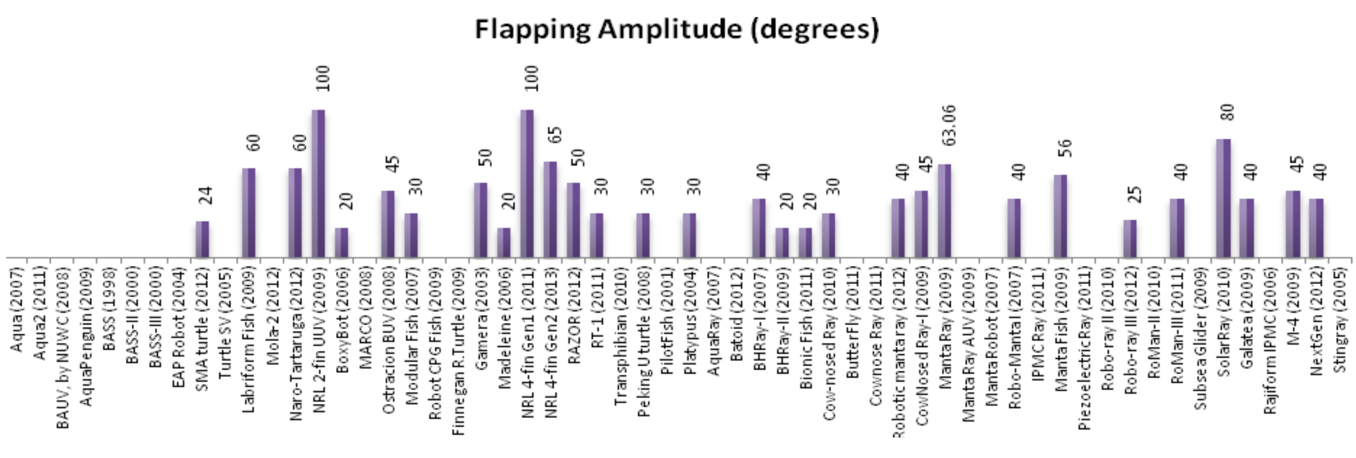

Fig. 13 A reference of UUV flapping fin amplitude. Only values determined to give the highest UUV velocity are listed.

REFERENCES

[1] C. Gans, A. S. Gaunt, and P. W. Webb, "Vertebrate Locomotion," in *Handbook of Physiology*, Ed. W. H. Dantzler, American Physiological Society, Oxford University Press, Oxford, UK, 1997, pp. 55-213.

[2] P. W. Webb, "Body form, locomotion and foraging in aquatic vertebrates," *American Zoologist*, vol. 24, pp. 107-120, 1984.

[3] P. Domenici, "Habitat, body design and the swimming performance of fish," in *Vertebrate Biomechanics and Evolution*, BIOS Scientific Publishers Ltd., Oxford, UK, 2003, pp. 137-160.

[4] K. Korsmeyer, J. Steffensen, and J. Herskin, "Energetics of median and paired fin swimming, body and caudal fin swimming, and gait transition in parrotfish (Scarus schlegeli) and triggerfish (Rhinecanthus aculeatus)," *The Journal of Experimental Biology*, vol. 205, pp. 1253-1263, 2002.

[5] F. Fish, "Biological Designs for Enhanced Maneuverability: Analysis of Marine Mammal Performance," *Proceedings of the Special Session on Bio-Engineering Research Related to Autonomous Underwater Vehicles*, 1997, pp. 109-117.

[6] R. W. Blake, "Fish functional design and swimming performance," *Journal of Fish Biology*, vol. 65, pp. 1193-1222, 2004.

[7] C. J. Fulton, "Swimming speed performance in coral reef fishes: field validations reveal distinct functional groups," *Coral Reefs*, vol. 26, pp. 217-228, 2007.

[8] C. J. Fulton and D. R. Bellwood, "Wave-induced water motion and the functional implications for coral reef fish assemblages," *Limnology and Oceanography*, vol. 50, pp. 255–264, 2005.

[9] J. L. Kendall, K. S. Lucey, E. A. Jones, J. Wang, and D. J. Ellerby, "Mechanical and energetic factors underlying gait transitions in bluegill sunfish (Lepomis macrochirus)," *Journal of Experimental Biology*, vol. 210, pp. 4265-4271, 2007.

[10] A. J. Murphy and M. Haroutunian, "Using BioInspiration to Improve Capabilities of Underwater Vehicles," *17th International Unmanned Untethered Submersible Technology Conference* (UUST), Portsmouth, NH, USA, 2011, pp. 20-31.

[11] I. C. Stobutzki and D. R. Bellwood, "An analysis of the sustained swimming abilities of pre- and post-settlement coral reef fishes," *Journal of Experimental Marine Biology and Ecology*, vol. 175, pp. 275-286, 1994.

[12] A. C. Gibb, B. C. Jayne, and G. V. Lauder, "Kinematics of pectoral fin locomotion in the bluegill sunfish Lepomis macrochirus," *Journal of Experimental Biology*, vol. 189, pp. 133-161, 1994.

[13] E. Drucker and J. Jensen, "Pectoral fin locomotion in the striped surfperch. I. Kinematic effects of swimming speed and body size," *The Journal of Experimental Biology*, vol. 199, no. 10, pp. 2235-2242, 1996.

[14] E. Drucker and J. Jensen, "Pectoral fin locomotion in the striped surfperch. II. Scaling swimming kinematics and performance at a gait transition," *The Journal of Experimental Biology*, vol. 199, no. 10, pp. 2243-2252, 1996.

[15] M. Mussi, A. P. Summers, and P. Domenici, "Gait transition speed, pectoral fin-beat frequency and amplitude in Cymatogaster aggregata, Embiotoca lateralis and Damalichthyes vacca," *Journal of Fish Biology*, vol. 61, pp. 1282-1293, 2002.

[16] M. Cannas, J. Schaefer, P. Domenici, and J. F. Steffensen, "Gait transition and oxygen consumption in swimming striped surfperch Embiotoca lateralis Agassiz," *Journal of Fish Biology*, vol. 69, no. 6, pp. 1612-1625, 2006.

[17] F. G. Whoriskey and R. J. Wootton, "The swimming endurance of threespine sticklebacks, Gasterosteus aculeatus L., from the Afon Rheidol, Wales," *Journal of Fish Biology*, vol. 30, no. 3, pp. 335-339, 1987.

[18] M. E. Hale, R. D. Day, D. H. Thorsen, and M. W. Westneat, "Pectoral fin coordination and gait transitions in steadily swimming juvenile reef fishes," *The Journal of Experimental Biology*, vol. 209, pp. 3708-3718.

[19] P. W. Webb, "The biology of fish swimming," in *Mechanics and Physiology of Animal Swimming*, Maddock L, Bone Q, Rayner JMV (eds), Cambridge University Press, Cambridge, 1994, pp. 45-62.

[20] J. J. Videler, "Swimming movements, body structure and propulsion in cod Gadus morhua," *Symposia of the Zoological Society of London*, no. 48, pp. 1-27, 1981.

[21] P. J. Geerlink, "The role of the pectoral fins in braking of mackerel, cod and saithe," *Netherlands Journal of Zoology*, vol. 37, pp. 81-104, 1987.

[22] P. W. Webb and A. G. Fairchild, "Performance and maneuverability of three species of teleostean fishes," *Canadian Journal of Zoology*, vol. 79, no. 10, pp. 1866-1877, 2001.

[23] P. W. Webb and V. De Buffrénil, "Locomotion in the biology of large aquatic vertebrates," *Transactions of the American Fisheries Society*, vol. 119, no. 4, pp. 629-641, 1990.

[24] J. E. Harris, "The role of the fins in the equilibrium of the swimming fish I. Wind-tunnel tests on a model of Mustelus canis (Mitchill)," *Journal of Experimental Biology*, vol. 13, no. 4, pp. 476-493, 1936.

[25] M. S. Gordon, J. R. Hove, P. W. Webb, and D. Weihs, "Boxfishes as unusually well controlled autonomous underwater vehicles," *Physiological and Biochemical Zoology*, vol. 73, no. 6, pp. 663-671, 2000.

[26] J. S. Palmisano, J. D. Geder, R. Ramamurti, M. Pruessner, W. C. Sandberg, and B. Ratna, "How to Maximize Pectoral Fin Efficiency through Control of Flapping Frequency and Bulk Rotation Amplitude," *18th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Portsmouth, NH, USA, 2013.

[27] F. Fish, "Swimming strategies for energy economy," *Fish swimming: an etho-ecological perspective*, pp. 90-122, 2010.

[28] E. J. Anderson, W. R. McGillis, and M. A. Grosenbaugh, "The boundary layer of swimming fish," *Journal of Experimental Biology*, vol. 204, pp. 81-102, 2001.

[29] F. E. Fish and J. T. Beneski, "Evolution and Bio-Inspired Design: Natural Limitations," in *Biologically Inspired Design*, Springer London, 2014, ch. 12, pp. 287-312.

[30] J. S. Palmisano, J. D. Geder, R. Ramamurti, W. C. Sandberg, and B. Ratna, "Robotic Pectoral Fin Thrust Vectoring Using Weighted Gait Combinations," *Applied Bionics and Biomechanics*, vol. 9, pp. 333-345, 2012.

[31] E. G. Drucker and G. V. Lauder, "Locomotor function of the dorsal fin in rainbow trout: kinematic patterns and hydrodynamic forces," *Journal of Experimental Biology*, vol. 208, no. 23, pp. 4479-4494, 2005.

[32] D. G. Logan, "Optimization of hybrid power sources for mobile robotics through the use of allometric design principles and dynamic programming," Ph.D. dissertation, Dept. Mech. and Nuclear Eng., The Pennsylvania State University, Pennsylvania, PA, 2010.

[33] J. D. Geder, R. Ramamurti, J. S. Palmisano, M. Pruessner, B. Ratna, and W. C. Sandberg, "Scaling Studies for an Actively Controlled Curvature Robotic Pectoral Fin," *International Conference on Intelligent Robotics and Applications* (ICIRA), Montreal, Canada, 2012.

[34] J. D. Geder, J. S. Palmisano, R. Ramamurti, M. Pruessner, B. Ratna, and W. C. Sandberg, "Biomimetic Design Process for an Underwater Flying and Hovering Vehicle," *Marine Technology Society Journal*, vol. 45, no. 4, pp. 74-82, 2011.

[35] J. D. Geder, R. Ramamurti, J. S. Palmisano, M. Pruessner, B. Ratna, and W. C. Sandberg, "Dynamic Performance of a Bio-Mimetic UUV: Effects of Fin Gaits and Orientation," *17th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Portsmouth, NH, USA, 2011.

[36] J. D. Geder, J. Palmisano, R. Ramamurti, W. C. Sandberg, and B. Ratna, "A New Hybrid Approach to Dynamic Modeling and Control Design for a Pectoral Fin Propelled UUV," *15th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Durham, NH, USA, 2007.

[37] J. D. Geder, J. Palmisano, R. Ramamurti, W. C. Sandberg, and B. Ratna, "Fuzzy Logic PID Based Control Design and Performance for Pectoral Fin Propelled Unmanned Underwater Vehicle," *International Conference on Control, Automation, and Systems*, Seoul, Korea, 2008.

[38] G. Dudek, P. Giguere, C. Prahacs, S. Saunderson, J. Sattar, L. Torres-Mendez, M. Jenkin, A. German, A. Hogue, A. Ripsman, J. Zacher, E. Milios, H. Liu, P. Zhang, M. Buehler, and C. Georgiades, "AQUA: An Amphibious Autonomous Robot," *Computer*, vol. 40, pp. 46-53, 2007.

[39] Adept MobileRobots. (2011, September 14). *Aqua2 Datasheet* [Online]. Available: http://www.mobilerobots.com/Libraries/Downloads/Aqua2-AQ2-RevA.sflb.ashx

[40] P. Bandyopadhyay, S. Singh, D. P. Thivierge, A. Annaswamy, H. Leinhos, A. Fredette, and D. Beal, "Synchronization of Animal-Mimetic Multiple High-Lift Fins in an Underwater Vehicle Using Olivo-Cerebellar Dynamics," *IEEE Journal of Oceanic Engineering*, vol. 33, no. 4, pp. 563-578, 2008.

[41] Festo AG & Co. KG. (2009, July 10). *AquaPenguin: A group of artificial autonomous penguins* [Online]. Available: http://www.festo.com/rep/en_corp/assets/pdf/AquaPenguin_en.pdf

[42] N. Kato, "Locomotion by mechanical pectoral fins," *Journal of Marine Science and Technology*, vol. 3, no. 3, pp. 113-121, 1998.

[43] N. Kato, "Control Performance in the Horizontal Plane of a Fish Robot with Mechanical Pectoral Fins," *IEEE Journal of Oceanic Engineering*, vol. 25, pp. 121-129, 2000.

[44] N. Kato, B. W. Wicaksono, and Y. Suzuki, "Development of biology-mimetic autonomous underwater vehicle 'BASS III' with high

maneuverability," *IEEE Proceedings of the 2000 International Symposium on Underwater Technology*, Tokyo, Japan, 2000, pp. 84-89.
[45] M. Anton, A. Punning, A. Aabloo, and M. Kruusmaa, "Towards a Biomimetic EAP Robot," *Proceedings of Towards the Autonomous Mobile Robots* (TAROS), Essex, UK, 2004.
[46] H. Kim, S. Song, and S. Ahn, "A turtle-like swimming robot using a smart soft composite (SSC) structure," *Smart Materials and Structures*, vol. 22, 2013.
[47] A. Konno, T. Furuya, A. Mizuno, K. Hishinuma, K. Hirata, and M. Kawada, "Development of Turtle-like Submergence Vehicle," *Proceedings of the 7th International Symposium on Marine Engineering*, Tokyo, Japan, 2005.
[48] P. Sitorus, Y. Nazaruddin, E. Leksono, and A. Budiyono, "Design and Implementation of Paired Pectoral Fins Locomotion of Labriform Fish Applied to a Fish Robot," *Journal of Bionic Engineering*, vol. 6, pp. 37-45, 2009.
[49] E. Ackerman. (2012, August 28). *AeroVironment's Mola Robot Flies Underwater on Solar Power* [Online]. Available: http://auvac.org/community-information/community-news/view/1749
[50] ETH Zürich Autonomous Systems Lab. (2012, November 21). *naro-nautical robot* [Online]. Available: http://www.naro.ethz.ch/p2/tartaruga.html
[51] J. D. Geder, R. Ramamurti, J. S. Palmisano, M. Pruessner, B. Ratna, and W. C. Sandberg, "Sensor Data Fusion and Submerged Test Results of a Pectoral Fin Propelled UUV," *16th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Durham, NH, USA, 2009.
[52] D. Lachat, A. Crespi, and A. J. Ijspeert, "BoxyBot: a swimming and crawling fish robot controlled by a central pattern generator," *International Conference on Biomedical Robotics and Biomechatronics* (BioRob), Pisa, Italy, 2006, pp. 643-648.
[53] P. Kodati, J. Hinkle, A. Winn, and X. Deng, "Microautonomous Robotic Ostraciiform (MARCO): Hydrodynamics, Design, and Fabrication," *IEEE Transactions on Robotics*, pp. 105-117, 2008.
[54] Barbera, Giovanni, Lijuan Pi, and X. Deng. "Attitude control for a pectoral fin actuated bio-inspired robotic fish," *IEEE International Conference on Robotics and Automation* (ICRA), 2011, pp. 526-531.
[55] Y. Hu, L. Wang, W. Zhao, Q. Wang, and L. Zhang, "Modular Design and Motion Control of Reconfigurable Robotic Fish," *46th IEEE Conference on Decision and Control*, New Orleans, LA, USA, 2007.
[56] M. Wang, J. Yu, and M. Tan, "Modeling neural control of robotic fish with pectoral fins using a CPG-based network," *Proceedings of the 48th IEEE Conference on Decision and Control*, held jointly with the *28th Chinese Control Conference* (CDC/CCC), Shanghai, China, 2009, pp. 6502-6507.
[57] S. Licht, M. Wibawa, F. Hover, and M. Triantafyllou, "Towards amphibious robots: Asymmetric flapping foil motion underwater produces large thrust efficiently," *16th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Durham, NH, USA, 2009.
[58] S. Hsu, C. Mailey, E. Eade, and J. Janet, "Autonomous control of a horizontally configured undulatory flap propelled vehicle," *IEEE International Conference on Robotics and Automation* (ICRA), vol. 2, Taipei, Taiwan, 2003, pp. 2194-2199.
[59] R. Moody, "The design, construction, and testing of a flexible fin propelled autonomous underwater vehicle," *MTS/IEEE OCEANS Conference and Exhibition*, Honolulu, HI, 2001, vol. 4, pp. 2703-2707.
[60] J. H. Long Jr., J. Schumacher, N. Livingston, and M. Kemp, "Four flippers or two? Tetrapodal swimming with an aquatic robot," *Bioinspiration & Biomimetics*, vol. 1, issue 1, pp. 20-29, 2006.
[61] J. D. Geder, J. S. Palmisano, R. Ramamurti, M. Pruessner, W. C. Sandberg, and B. Ratna, "Four-Fin Bio-Mimetic UUV: Modeling and Control Solutions," *ASME 2011 International Mechanical Engineering Congress & Exposition* (IMECE), Denver, Colorado, USA, 2011.
[62] J. D. Geder, J. S. Palmisano, R. Ramamurti, and M. Pruessner, "Maneuvering Performance of a Four-Fin Bio-Inspired UUV," *OCEANS '13 MTS/IEEE*, San Diego, CA, USA, 2013.
[63] D. N. Beal, H. A. Leinhos, A. R. Fredette, and R. Berube, "Unified Scaling for Flapping Fins," *IEEE Journal of Oceanic Engineering*, vol. 38, no. 1, pp. 1-11, 2013.
[64] N. Kato, "Swimming and Walking of an Amphibious Robot With Fin Actuators," *The Marine Technology Society Journal*, vol. 45, no. 4, pp. 181-197, 2011.
[65] T. Kentaro, M. Yamamura, Z. Luo, M. Onishi, S. Hirano, K. Asaka, and Y. Hayakawa, "Development of a rajiform swimming robot using ionic polymer artificial muscles," *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Beijing, China, 2006, pp. 1861-1866.
[66] Y. Toda, M. Danno, K. Sasajima, and H. Miki, "Model Experiments on the Squid-Like Under-Water Vehicle with Two Undulating Side Fins," *Proceedings of the 4th International Symposium on Aero Aqua Bio-Mechanisms*, Shanghai, China, 2009, no. 45.
[67] S. Licht, "Biomimetic Robots For Environmental Monitoring In The Surf Zone & In Very Shallow Water," *IEEE/RSJ International Conference on Intelligent Robots and Systems* (IROS), Vilamoura, Algarve, Portugal, 2012.
[68] W. Zhao, Y. Hu, L. Wang, and Y. Jia, "Development of a flipper propelled turtle-like underwater robot and its CPG-based control algorithm," *47th IEEE Conference on Decision and Control*, Cancun, Mexico, 2008, pp. 5226-5231.
[69] B. Hobson, M. Kemp, and A. Leonessa, "Integration of a hovering module with the Morpheus AUV: application to MCM missions," *MTS/IEEE OCEANS '02*, Biloxi, MS, 2002, vol. 1, pp. 207-209.
[70] M. Kemp, B. Hobson, J. Janet, C. Pell, and E. Tytell, "Assessing the performance of oscillating fin thruster vehicles," *12th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Durham, NH, 2001.
[71] B. Hobson, M. Murray, and C. A. Pell, "PilotFish: Maximizing agility in an unmanned-underwater vehicle," *11th International Symposium on Unmanned Untethered Submersible Technology* (UUST), Durham, NH, USA, 1999, pp. 41-51.
[72] M. Basgall. (2001, June 29). *Swimming with Gamera* [Online]. Available: http://today.duke.edu/2001/06/gamera629.html
[73] N. Kato, J. Ayers, and H. Morikawa, "Biology-mimetic precision maneuvering of underwater vehicles," in *Bio-mechanisms of Swimming and Flying*, chap. 9, Springer Japan, 2008, pp. 111-125.
[74] Festo AG & Co. KG. (2007, April 3). *Aqua_ray: Water-hydraulic manta ray with flapping-wing drive* [Online]. Available: http://www.festo.com/rep/en_corp/assets/pdf/Aqua_ray_en.pdf
[75] A. Cloitre, V. Subramaniam, N. Patrikalakis, and P. Valdivia y Alvarado, "Design and control of a field deployable batoid robot," *4th IEEE RAS & EMBS International Conference on Biomedical Robotics and Biomechatronics* (BioRob), Rome, Italy, 2012, pp. 707-712.
[76] J. Gao, S. Bi, Y. Xu, and C. Liu, "Development and design of a robotic manta ray featuring flexible pectoral fins," *IEEE International Conference on Robotics and Biomimetics* (ROBIO), Sanya, China, 2007, pp. 519-523.
[77] J. Gao, S. Bi, J. Li, and C. Liu, "Design and experiments of robot fish propelled by pectoral fins," *IEEE International Conference on Robotics and Biomimetics* (ROBIO), Guilin, Guangxi, China, 2009, pp. 445-450.
[78] Y. Cai, S. Bi, K. H. Low, L. Zhang, and G. Zong, "Posture analysis and application of a bionic pectoral foil," *IEEE International Conference on Robotics and Biomimetics* (ROBIO), Phuket, Thailand, 2011, pp. 1783-1788.
[79] Y. Cai, S. Bi, and L. Zhang, "Design and implication of a bionic pectoral fin imitating cow-nosed ray," *IEEE/RSJ International Conference on Intelligent Robots and Systems*, Taipei, Taiwan, 2010.
[80] L. Shi, S. Guo, and K. Asaka, "A novel butterfly-mimetic underwater microrobot with pectoral fins," *International Conference on Mechatronics and Automation* (ICMA), Beijing, China, 2011, pp. 853-858.
[81] Z. Chen, T. I. Um, J. Zhu, and H. Bart-Smith, "Bio-Mimetic Robotic Cownose Ray Propelled By Electroactive Polymer Pectoral Fin," *Proceedings of the ASME 2011 International Mechanical Engineering Congress & Exposition* (IMECE), Denver, Colorado, USA, 2011.
[82] Z. Chen, T. I. Um, and H. Bart-Smith, "Bio-mimetic Robotic Manta Ray Powered by Ionic Polymer-Metal Composite Artificial Muscles," *International Journal of Smart and Nano Materials*, 2012.
[83] S. Yang, J. Qiu, and X. Han, "Kinematics Modeling and Experiments of Pectoral Oscillation Propulsion Robotic Fish," *Journal of Bionic Engineering*, vol. 6, pp. 174-179, 2009.
[84] Z. Wang, Y. Wang, J. Li, and G. Hang, "A micro biomimetic manta ray robot fish actuated by SMA," *IEEE International Conference on*

Robotics and Biomimetics (ROBIO), Guilin, Guangxi, China, 2009, pp. 1809-1813.

[85] Y. Zhong, D. Zhang, C. Zhou, C. W. Chong, T. Hu, L. Shen, and K. H. Low, "Better endurance and load capacity: An underwater vehicle mimetic by manta ray," *4th International Symposium on Aero Aqua Bio-Mechanisms* (ISABMEC), Shanghai, China, 2009.

[86] K. Suzumori, S. Endo, T. Kanda, N. Kato, and H. Suzuki, "A Bending Pneumatic Rubber Actuator Realizing Soft-bodied Manta Swimming Robot," *IEEE International Conference on Robotics and Automation* (ICRA), Rome, Italy, 2007, pp. 4975-4980.

[87] Y. Xu, G. Zong, S. Bi, and J. Gao, "Initial development of a flapping propelled unmanned underwater vehicle (UUV)," *IEEE International Conference on Robotics and Biomimetics* (ROBIO), Sanya, China, 2007, pp. 524-529.

[88] Y. Cai, S. Bi, L. Zhang, and J. Gao, "Design of a robotic fish propelled by oscillating flexible pectoral foils," *IEEE/RSJ International Conference on Intelligent Robots and Systems* (IROS), St. Louis, MO, USA, 2009, pp. 2138-2142.

[89] W. Zhao, T. Osaka, A. Ming, and M. Shimojo, "Development of a Soft Underwater Robot Mimicking Cow-nosed Ray," *IEEE International Conference on Robotics and Biomimetics* (ROBIO), Phuket, Thailand, 2011.

[90] Y. Cai, S. Bi, and L. Zheng, "Design and Experiments of a Robotic Fish Imitating Cow-Nosed Ray," *Journal of Bionic Engineering*, vol. 7, pp. 120-126, 2010.

[91] C. Niu, L. Zhang, S. Bi, and Y. Cai, "Development and Depth Control of a Robotic Fish Mimicking Cownose Ray," *Proceedings of the 2012 IEEE International Conference on Robotics and Biomimetics* (ROBIO), Guangzhou, China, 2012, pp. 814-818.

[92] C. Zhou and K. H. Low, "Better endurance and load capacity: An improved design of manta ray robot (RoMan-II)," *Journal of Bionic Engineering*, vol. 7, pp. S137-S144, 2010.

[93] K. H. Low, C. Zhou, G. Seet, S. Bi, and Y. Cai, "Improvement and testing of a robotic manta ray (RoMan-III)," *IEEE International Conference on Robotics and Biomimetics* (ROBIO), Phuket, Thailand, 2011, pp. 1730-1735.

[94] Evo Logics. (2009, March 27). *Subsea Glider with Fin Ray Effect* [Online]. Available: http://www.evologics.de/en/products/glider/index.html

[95] R. Love, A. Arroyo, and E. Schwartz, "Solar Ray: An Autonomous Solar-Powered Biomimetic Flapping-Wing Underwater Vehicle," *Florida Conference on Recent Advances in Robotics*, Jacksonville, FL, USA, 2010.

[96] D. G. Simons, M. M. C. Bergers, S. Henrion, J. I. J. Hulzenga, R. W. Jutte, W. M. G. Pas, M. van Schravendijk, T. G. A. Vercruyssen, and A. P. Wilken, "A highly versatile autonomous underwater vehicle with biomechanical propulsion," *OCEANS 2009 - EUROPE*, Bremen, Germany, 2009, pp. 1-6.

[97] A. Willy and K. H. Low, "Initial experimental investigation of undulating fin," *IEEE/RSJ International Conference on Intelligent Robots and Systems* (IROS), Edmonton, Alberta, Canada, 2005, pp. 1600-1605.

[98] J. A. Walker and M. W. Westneat, "Performance limits of labriform propulsion and correlates with fin shape and motion," *Journal of Experimental Biology*, vol. 205, no. 2, pp. 177-187, 2002.

[99] S. Gibson and I. A. Johnston, "Scaling relationships, individual variation and the influence of temperature on maximum swimming speed in early settled stages of the turbot Scophthalmus maximus," *Marine Biology*, vol. 121, no. 3, pp. 401-408, 1995.

[100] J. Davenport, S. A. Munks, and P. J. Oxford, "A Comparison of the Swimming of Marine and Freshwater Turtles," *Proceedings of the Royal Society of London. Series B, Biological Sciences*, vol. 220, no. 1221, pp. 447-475, 1984.

[101] E. A. Jones, K. S. Lucey, and D. J. Ellerby, "Efficiency of labriform swimming in the bluegill sunfish (Lepomis macrochirus)," *Journal of Experimental Biolology*, vol. 210, pp. 3422-3429, 2007.

[102] K. Sato, Y. Watanuki, A. Takahashi, P. J. O. Miller, H. Tanaka, R. Kawabe, P. J. Ponganis, Y. Handrich, T. Akamatsu, Y. Watanabe, Y. Mitani, D. Costa, C. Bost, K. Aoki, M. Amano, P. Trathan, A. Shapiro, and Y. Naito, "Stroke frequency, but not swimming speed, is related to body size in free-ranging seabirds, pinnipeds and cetaceans," *Proceedings of the Royal Society B: Biological Sciences*, vol. 274, no. 1609, pp. 471-477, 2007.

[103] C. C. Yığın and A. Ismen, "Length-weight relationships for seven rays from Saros Bay (North Aegean Sea)," *Journal of Applied Ichthyology*, vol. 25, no. s1, pp. 106-108, 2009.

[104] S. G. Raje, "Length-weight relationship of five species of rays from Mumbai, Maharashtra," *Indian Journal of Fisheries*, vol. 47, no. 2, pp. 159-161, 2011.

[105] H. Filiz and S. Mater, "A preliminary study on length-weight relationships for seven elasmobranch species from North Aegean Sea, Turkey," *EÜ Su Ürünleri Dergisi*, vol. 19, no. 3-4, pp. 401-409, 2002.

[106] K. A. Coull, "Length/weight relationships for 88 species of fish encountered in the North East Atlantic," Department of Agriculture and Fisheries for Scotland, Scottish Fisheries Research Report, no. 43, 1989.

[107] K. F. Lagler and V. C. Applegate, "Relationship between the length and the weight in the snapping turtle Chelydra serpentina Linnaeus," *The American Naturalist*, vol. 77, no. 772, pp. 476-478, 1943.

[108] W. A. Dunson, "Relationship between Length and Weight in the Spiny Softshell Turtle," *Copeia 1967*, no. 2, pp. 483-485, 1967.

[109] T. T. Jones, M. D. Hastings, B. L. Bostrom, D. Pauly, and D. R. Jones, "Growth of captive leatherback turtles, Dermochelys coriacea, with inferences on growth in the wild: Implications for population decline and recovery," *Journal of Experimental Marine Biology and Ecology*, vol. 399, pp. 84-92, 2011.

[110] M. Deflorio, A. Aprea, A. Corriero, N. Santamaria, and G. De Metrio, "Incidental captures of sea turtles by swordfish and albacore longlines in the Ionian sea," *Fisheries Science*, vol. 71, no. 5, pp. 1010-1018, 2005.

[111] T. E. Graham and T. S. Doyle, "Growth and Population Characteristics of Blanding's Turtle, Emydoidea blandingii, in Massachusetts," *Herpetologica*, pp. 410-414, 1977.